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A GENERALIZED MATHEMATICAL SOLUTION FOR TRANSIENT OFF TRACKING OF SINGLE VEHICLES AND TRUCK COMBINATIONS

J.H.F. Woodrooffe, L.E. Morisset, C.A.M. Smith

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A GENERALIZED MATHEMATICAL SOLUTION FOR TRANSIENT OFF TRACKING OF SINGLE VEHICLES AND TRUCK COMBINATIONS

SOLUTION MATHÉMATIQUE GÉNÉRALE POUR LE DÉPORTEMENT EN RÉGIME TRANSITOIRE DE VÉHICULES OU DE COMBINAISONS DE VÉHICULES

J.H.F. Woodrooffe, L.E. Morisset, C.A.M. Smith

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ABSTRACT

A simplified mathematical solution to predict transient off tracking of multiple vehicle combinations is presented. The generalized equations assume a constant radius curve of any angular duration with a tangent lead in and run out. The validated solution yields the off tracking of a vehicle or vehicle combination with respect to a track mounted observer. The equations are of continuous piecewise form and are field solvable.

RÉSUMÉ

Une solution mathématique simplifiée pour le déportement en régime transitoire de diverses combinaisons de véhicules est présentée. Les équations générales supposent un rayon de courbure constant sur un angle quelconque avec des sections d'accès et une sortie tangente à la courbe. La solution vérifiée donne le déportement d'un véhicule ou d'une combinaison de véhicules par rapport à un observateur placé sur la trajectoire. Les équations sont de forme continue par intervalles et peuvent être résolues sur le terrain.

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A GENERALIZED MATHEMATICAL SOLUTION FOR TRANSIENT OFF TRACKING OF SINGLE VEHICLES AND TRUCK COMBINATIONS

1.0 INTRODUCTION

1.1 General

The subject of vehicle off tracking can be split into two specific categories; steady off tracking and transient off tracking. In this report transient off tracking refers to the off tracking of a vehicle that occurs throughout a specific curve of some defined radius and angular displacement assuming a tangent lead in and run out. An exanple of this is the common 90 degree curve at a roadway intersection.

A mathematical solution to steady state off tracking of multi unit vehicle commonly referred to as the "sum of squares method" has been developed by the Western Highway Institute [Ref. 1]. Contained in the SAE Handbook [Ref. 2] are generalized tables and graphs which simplify the required calculation and yield approximate steady state off tracking values. Graphics construction techniques are also commonly used to determine the steady state off tracking of multi unit vehicles in constant radius curves.

Few previous works focusing on the analytical approach to the problem of transient off tracking are available. In 1938, W. Schaar [Ref. 3] formulated what appears to be the first analytical solution to transient off tracking of a single vehicle. Two other authors, Fazekas [Ref. 4] and Jindra [Ref. 5], have also studied transient off tracking and have presented analytical solutions for single vehicles with an approximate extension to double vehicle combinations such as the common semi-trailer. These solutions are quite complex and not readily extendable. Stevens et al [Ref. 6] presented charts for off tracking calculations for 90 and 270 degree turns and underline the need for a more general off tracking solution.

1.2 Intent

The intent of this report is to present the mathematical arguments leading to a convenient solution to transient off tracking of vehicle combinations aimed specifically at the practicing engineer. Attempts have been made to limit the complexity of the solution development so that it approximates test results within practical engineering limits. To this extent, the solution can be thought of as an argument of close approximation rather than a precise mathematical exercise.

The solution is hybrid in nature, utilizing the existing, proven steady state off tracking theory [Refs. 1, 2], together with a decay function which is the focal point of the mathematical derivation presented here. It is hoped that road and vehicle designers and government compliance experts can benefit from this solution.

1.3 Report Structure

The body of the report contains the mathematical derivation of the solution supported by various graphs linking the theory with experimental data for validation purposes. Section 2.4 contains a summary of the equations which are then applied in Appendix A to solve for a range of "real life" off tracking cases. This is intended for those interested in applying the equation and not in the details of the derivation. Appendix D contains a simple computer program written in 'Basic' which permits quick calculations of transient off tracking of single or multiple unit vehicles of up to four trailers in

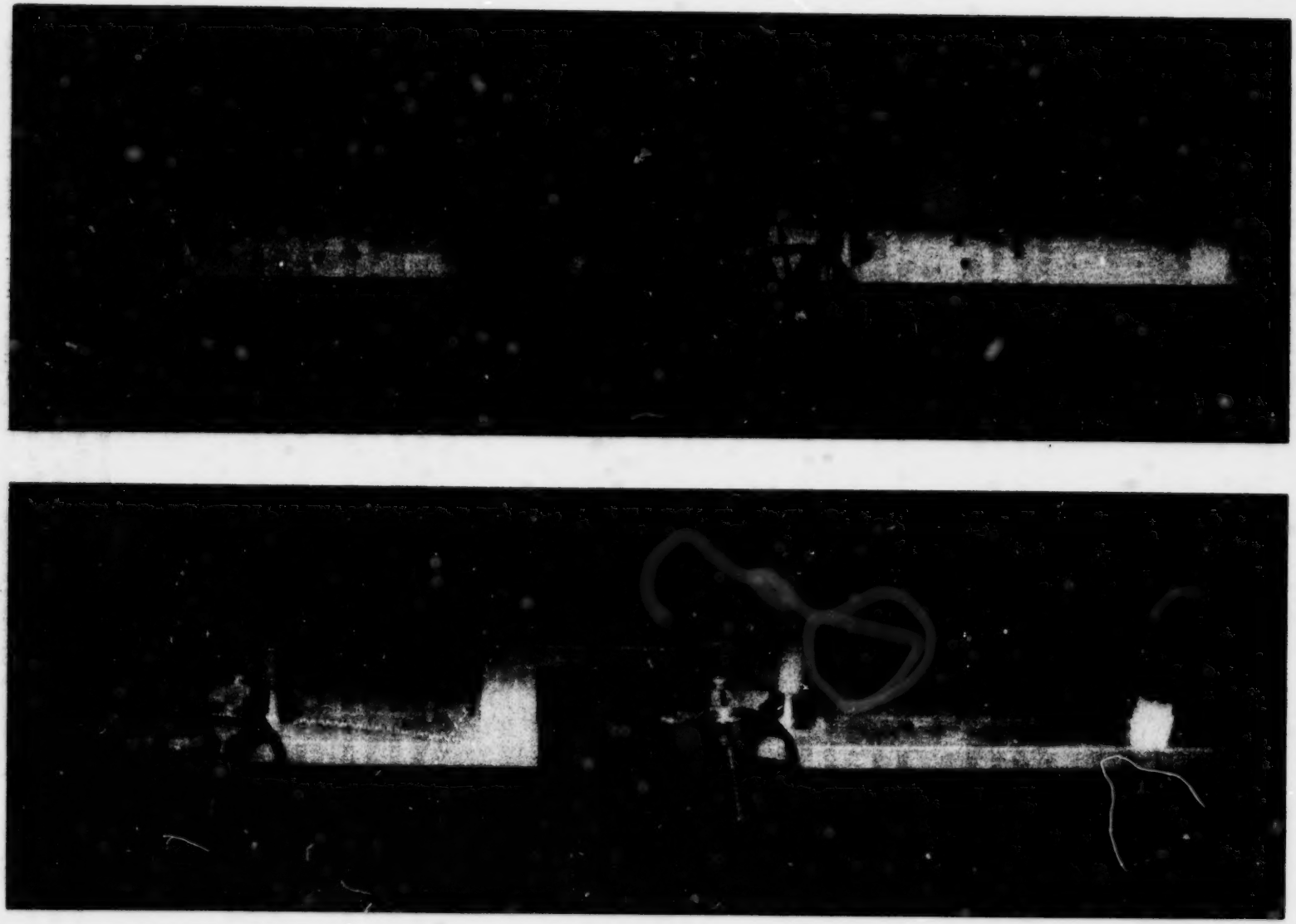

FIG. 1: A-TRAIN ABOVE, B-TRAIN BELOW

length, coupled as A-trains (multiple full trailers), B-trains (multiple semi-trailers), and C-trains (multiple trailers equipped with B-dollies); with or without stingers. The road alignment (path) used in the program is a tangent (straight) lead in to a constant radius curve of any radius and duration ending with a tangent exit. A higher level interactive program with a plotting routine has also been developed and is presented in Reference 7.

1.4 Experimental Data Sources

To generate the experimental data required during the analysis, two sources were used. One being a scale model of a transporter which consisted of two units of identical length representing trailers and a shorter unit representing a detachable converter dolly. The other source was field data from full size truck tests. By including the dolly in the consist, the configuration represents an A-train. The B-train configuration is achieved by excluding the dolly from the consist and attaching the following trailer to the lead trailer. Photographs of the scale model trailers in these two configurations are shown in Figure 1.

A pen is located at the centre of the trailing axle of each unit to scribe a tracking line on a layout sheet initially marked with the steering path (alignment). The measured off tracking was defined as the length of a line drawn perpendicular to the original alignment, intersecting the ink traces of the trailers.

Representation of a vehicle configuration of more than two units was achieved by following the trailing ink traces of the previous unit or units. For example, to model a triple B-train, the lead unit of a two vehicle consist representing the first two trailers, would follow the alignment path leaving the off tracking ink trace of the second unit on the layout sheet. Using this off tracking trace as its path, the remaining single unit representing the last of the three trailers would be led through the curve tracing its off tracking path. Drawings of the models can be found in Appendix B.

Full scale off tracking data from previous field experiments were also used for validation of the relationship. This is outlined in detail in Section 3.0 and Appendix C.

The following is a list of defined terms which will appear in the development of the off tracking equations.

1.5 Term Definitions

Alignment path — the path that the front axle centre of the vehicle follows, i.e. the road shape.

Curve commencement — corresponds to the intersecting point of the entry tangent with the circular curve.

Distance travelled (s) — the distance travelled along the alignment path, where s = 0 at curve commencement. Distance is considered negative prior to the curve commencements.

Maximum off tracking (0Tmax) — the maximum off tracking achieved by a given vehicle or vehicle combination through a particular curve. In curves of short duration this need not be the steady state value.

Negative off tracking — also referred to as out tracking. A condition where the trailing axle centre is outside the alignment curve such as when a vehicle progresses from a tangent alignment condition, assuming that the continuous circle is considered to be the principle alignment. The method of measurement conforms with that defined for conventional off tracking.

Off tracking (OT) — is a phenomenon where the trailing axles of a vehicle migrate toward the curve centre until it reaches a steady state offset from the alignment.

The measured quantity of off tracking is defined as the radial distance separating the rear axle centre path from the front axle centre alignment path.

On curve alignment — an initial condition where all axle centres of a vehicle lie on the same constant radius curve.

Pintle hook — the hitch mechanism located on the lead unit which captures the tow eyes of the following unit.

Rear overhang — the rear overhang of a truck, semi-tractor or full trailer is the distance between the centreline of the vehicle's rear axle group centre, and the centreline of its pintle hook.

Steady state off tracking (OT_{ss}) — a condition where a projected extension of all fixed axles of a vehicle pass through the curve centre forming a right angle triangle with the vehicle where the hypotenuse is the alignment curve radius and the right angle is formed by the vehicle wheelbase and the radius of curvature at the trailing axle centre (see Fig. 5).

Stinger — a rear extension of the lead trailer or track to which is mounted the coupled mechanism (usually a pintle hook) for the following trailer.

Tangent alignment — an initial condition where all axle centres of a vehicle lie on an entry tangent to a curve with the front axle of the vehicle positioned at the curve commencement ($s = 0$).

Truck trailer — a truck trailer is a vehicle with or without auxiliary motive power designed to be drawn by a motor truck or truck tractor.

Figure 2 has been prepared to help visualize the types of vehicles herein defined. It is based on the use of the digit system of identifying combinations by axles. In this system of identification, each digit indicates the number of axles on a vehicle which is entirely supported on its own wheels, such as motor truck or full trailer. The first digit refers to the power unit and a second single digit refers to a full trailer, while a second digit with an S prefix refers to a semi-trailer. For example:

2-S2 = 2-axle truck tractor and 2-axle semi-trailer
2-1 = 2-axle truck with 1-axle full trailer
3-S2-2 = 3-axle truck tractor, 2-axle semi-trailer, and 2-axle full trailer, [Ref. 2].

Full trailer — a full trailer is a truck trailer constructed so that all its own weight and that of its load rests upon its own wheels. (See Fig. 2, types 2-1, and 3-S2-2.) [Ref. 2]

Semi-trailer — a semi-trailer is a truck trailer equipped with one or more axles, and so constructed that the front end and a substantial part of its own weight and that of its load rests upon another vehicle. (See Fig. 2, type 2-S2.) [Ref. 2]

Trailer converter dolly — a trailer converter dolly is a very short wheelbase truck trailer with one or more axles equipped with a fifth wheel, a drawbar, and other parts necessary to convert a semi-trailer to a full trailer. [Ref. 2]

Turning centre — is that point about which all parts of a vehicle or combination of vehicles revolve in describing a turn of constant radius and to which all wheel spindles are normally radial. In the case of two-axled bogies or tandems in which the axles are constrained to parallelism, the interaxle trunnion or its equivalent is assumed to be radial from this point. (See Fig. 2) [Ref. 2]

Wheelbase — the distance between the lead axle and the trailing axle of a vehicle or vehicle unit. In the case of a tandem or tri-axle group, wheelbase is measured with respect to the geometrical centre of the group. For the purposes of off tracking, self steering axles are not recognized as part of an axle group. (See Appendix A.)

Equivalent wheelbase — is the theoretical wheelbase of a single vehicle having the identical steady state off tracking value as the vehicle combination in question.

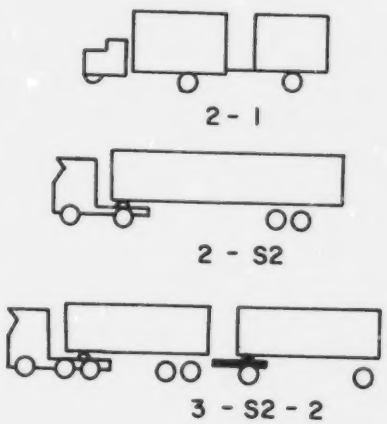

FIG. 2: SAE SYSTEM OF TRUCK IDENTIFICATION

2.0 DEVELOPMENT OF EQUATIONS

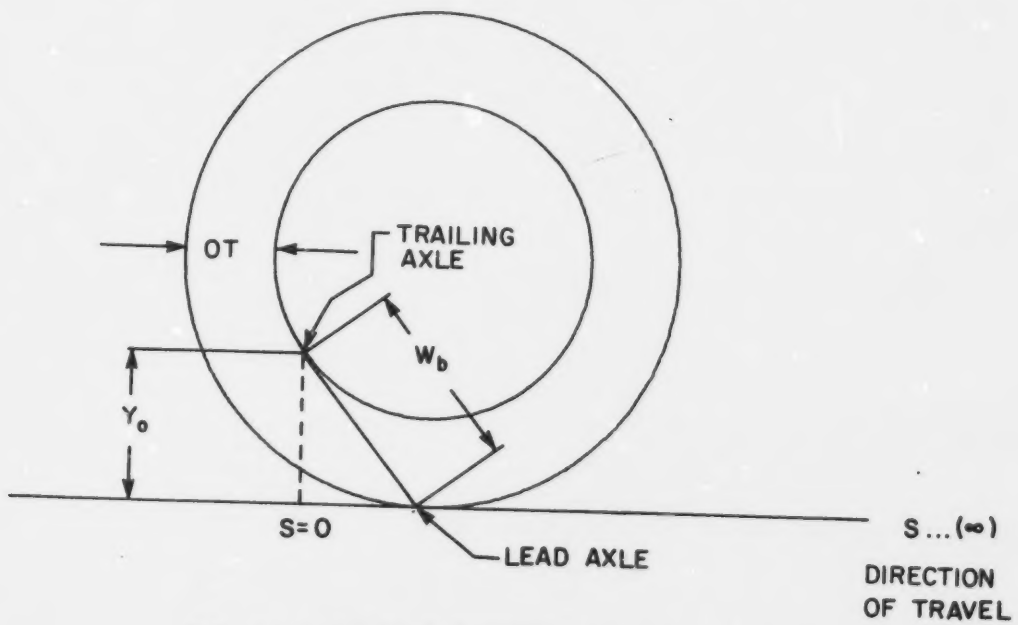

FIG. 3: VEHICLE POSITIONED FOR TANGENT EXIT

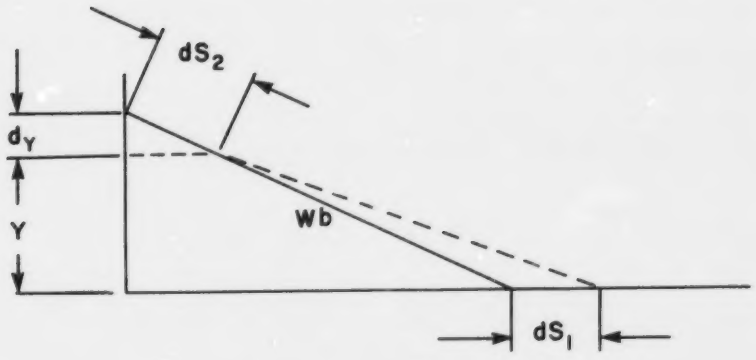

FIG. 4: RECTANGULAR CO-ORDINATE REPRESENTATION OF EXIT CONDITION

2.1 Curve Exit

Consider a single vehicle in a constant radius curve with the lead axle position on the exit tangent as shown in Figure 3.

It has been observed that the off tracking response of the vehicle as it negotiates the exit tangent can be represented on a rectangular co-ordinate system as shown in Figure 4.

Using infinitesimal calculus and simple triangular relations shown in Figure 4, and if we assume that $ds_1 \approx ds_2$ then

$$\frac{dy}{y} = -\frac{ds}{wb} \tag{1}$$

where the minus sign denotes decreasing off tracking.

Integrating Equation (1) to solve for y yields the following relationship

$$\int_{y_o}^{y} \frac{dy}{y} = -\frac{s}{wb}\Bigg|_{s_o}^{s}$$

$$\ell n \frac{y}{y_o} = -\left(\frac{s-s_o}{wb}\right)$$

hence

$$y = y_o \exp\left(-\left(\frac{s-s_o}{wb}\right)\right) \tag{2}$$

Given an initial condition where $s_o = 0$, the solution reduces to

$$y = y_o \exp\left(-\frac{s}{wb}\right) \tag{3}$$

where s is the distance travelled along the alignment path

y_o is the initial lateral displacement of the vehicle's trailing axle with respect to the tangent

y is the lateral displacement of the vehicle's trailing axle with respect to the tangent at any point s.

To extend this relationship to cover vehicle combinations, the wheelbase term wb can be replaced by the equivalent wheelbase term wb_e. Equivalent wheelbase is defined as the theoretical wheelbase of a single vehicle having the identical steady state off tracking value OT_{ss} as the vehicle combination being modelled. For an n vehicle combination, the equivalent wheelbase may be found in the following manner.

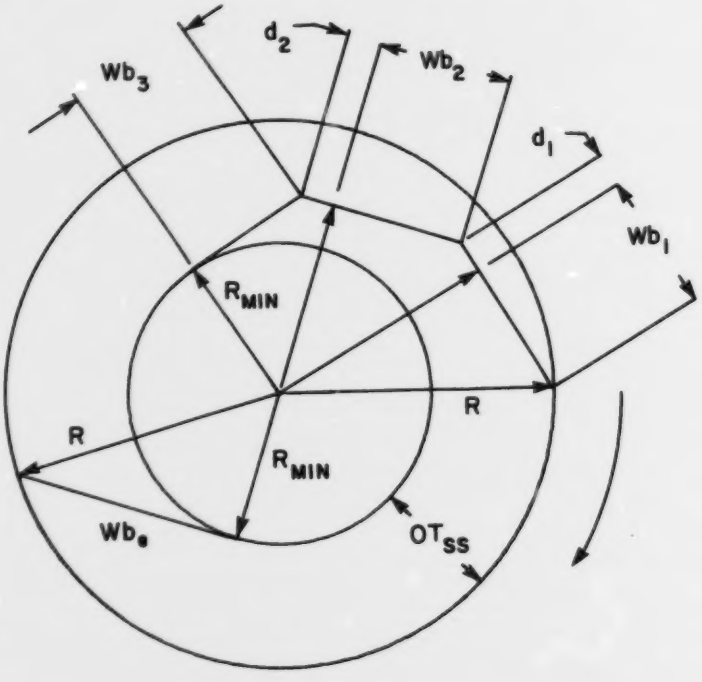

FIG. 5: A THREE-UNIT ARTICULATED VEHICLE REPRESENTED BY A SINGLE UNIT VEHICLE wb_e

Referring to Figure 5, for an n vehicle combination, the equivalent wheelbase may be found in the following manner

$$wb_e = \sqrt{R^2 - (R - OT_{ss})^2} \tag{4}$$

$$R - OT_{ss} = \sqrt{R^2 - \left(\sum_{i=1}^{n} wb_i^2\right) + \left(\sum_{i=1}^{n-1} d_i^2\right)} \tag{5}$$

thus

$$wb_e = \sqrt{\sum_{i=1}^{n} wb_i^2 - \sum_{i=1}^{n-1} d_i^2} \tag{6}$$

where d represents the rear overhang of a vehicle measured from the trailing axle group centre to the attachment point of the following trailer. Note the overhang of the last trailer is not included in the summation $\sum_{i=1}^{n-1} d_i^2$.

With the derivation of wb_e the general equation for any number of vehicles given that $wb_e < R$, is

$$y = y_o \exp\left(-\frac{s}{wb_e}\right) \tag{7}$$

By definition, off tracking (OT) is the perpendicular distance from the alignment path to the path of the trailing axle centre as shown by line AB in Figure 6. The maximum off tracking attained before exit is denoted by OT_{max} and in this case would be distance AB located at point B. From point B to infinity it is clear that the off tracking would diminish to zero.

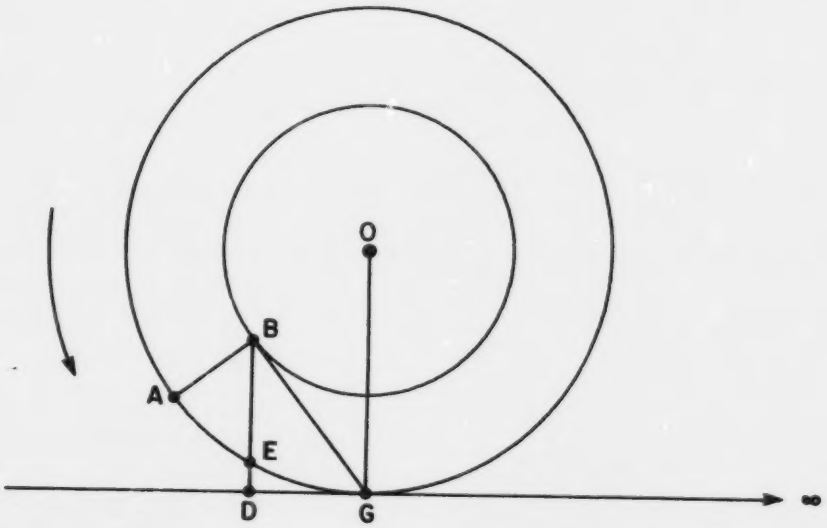

FIG. 6: ILLUSTRATION OF MAXIMUM OFF TRACKING TERMS AT POINT OF EXIT

While the trailing axle is in the sector bounded by arc length AG the off tracking (OT) is measured with respect to a curvilinear co-ordinate. As the rear axle centre crosses the radial projection OG at the curve to tangent intersection, the remaining off tracking is measured with respect to a rectangular co-ordinate system. The presence of these two co-ordinate systems precludes the use of a simple single equation to describe exactly this portion of the exit phenomena. However, for practical purposes, a solution based upon the rectangular co-ordinate will adequately describe the off tracking response of a vehicle as the error incurred in sector AB, (Fig. 6) is small.

The following rational was used for this part of the solution. With reference to Figure 6, it has been shown that from point B to infinity vehicle off tracking can be accurately described by Equation (7). This requires, however, that the decay term operates on the constant y_o which is an impractical value because within the sector bounded by points AG the result is not off tracking as previously defined. For practical reasons therefore, it is prudent to operate the decay term on the constant OT_{max} (line AB) rather than y_o (line BD). To accomplish this a displacement constant must be inserted into the exponent to shift the function backwards so that the decay function containing OT_{max} equals the decay function containing y_o at the point where the trailing axle of the vehicle reaches the tangent commencement line OG, (Fig. 6).

The modified equation now takes for form of

$$OT = OT_{max} \exp\left(-\left(\frac{s-c}{wb_e}\right)\right) \tag{8}$$

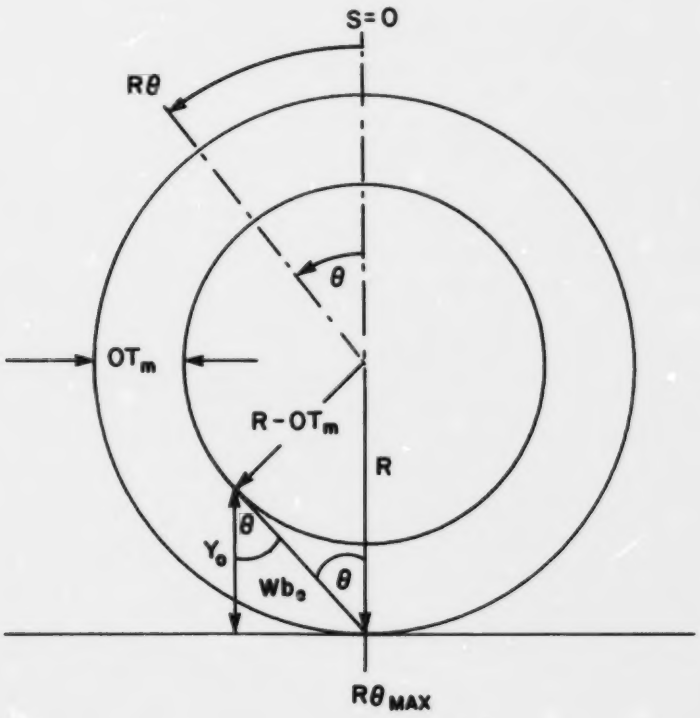

FIG. 7: GEOMETRIC ILLUSTRATION FOR DETERMINING THE CONSTANT C

The constant c can be determined in the following manner.

In reference to Figure 7, given that

$$(R - OT_{max}) = \sqrt{R^2 + wb_e^2 - 2Rwb_e \cos\phi} \tag{9}$$

and

$$y_o = wb_e \cos\phi \tag{10}$$

Isolating ϕ in both equations and solving simultaneously

$$y_o = \frac{R^2 + wb_e^2 - (R - OT_m)^2}{2R} \tag{11}$$

Substituting Equation (11) into Equation (7)

$$y = \frac{R^2 + wb_e^2 - (R - OT_m)^2}{2R} \exp\left(-\frac{s}{wb_e}\right) \tag{12}$$

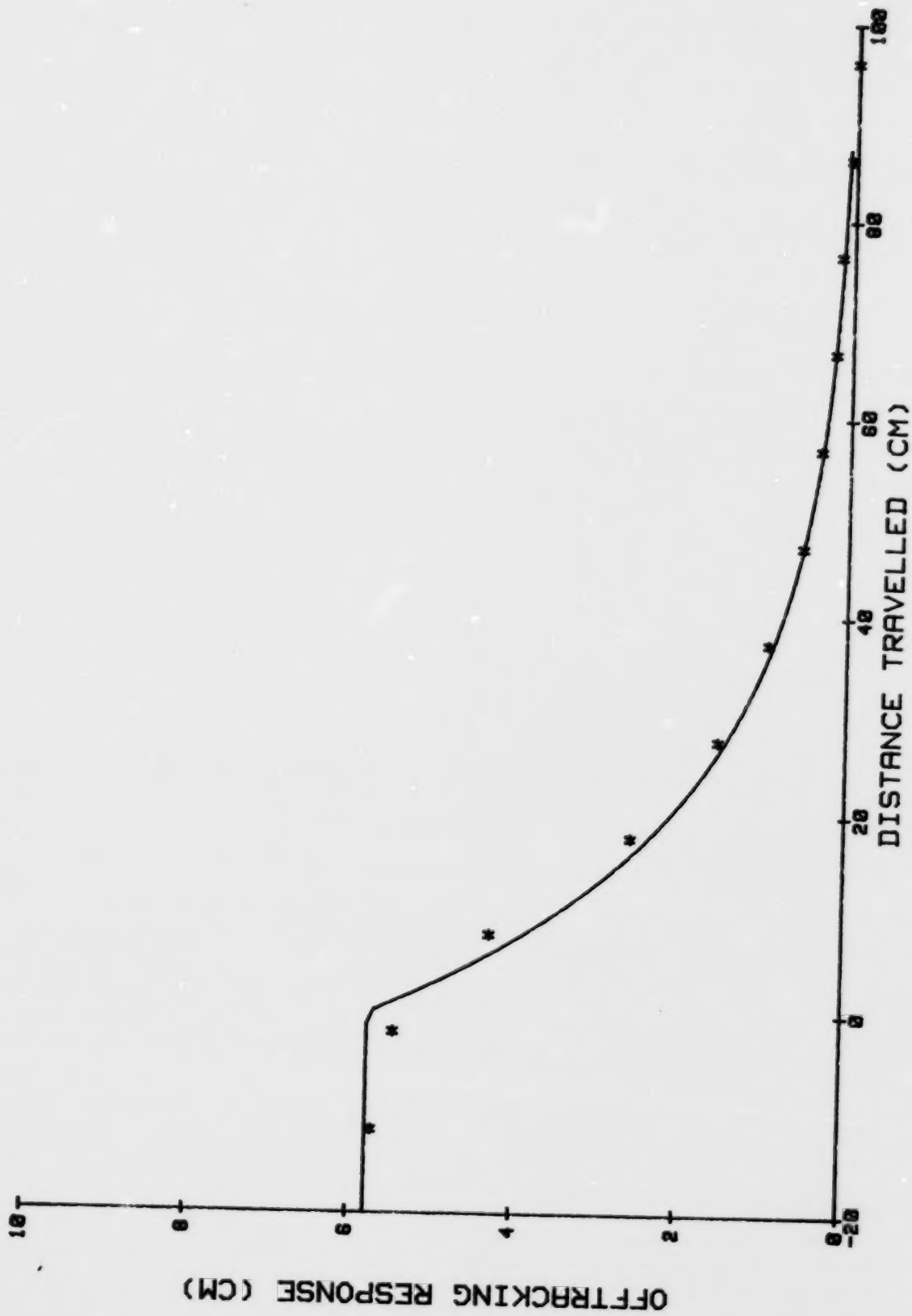

FIG. 8: MODEL TEST – SINGLE UNIT EXIT FROM STEADY STATE

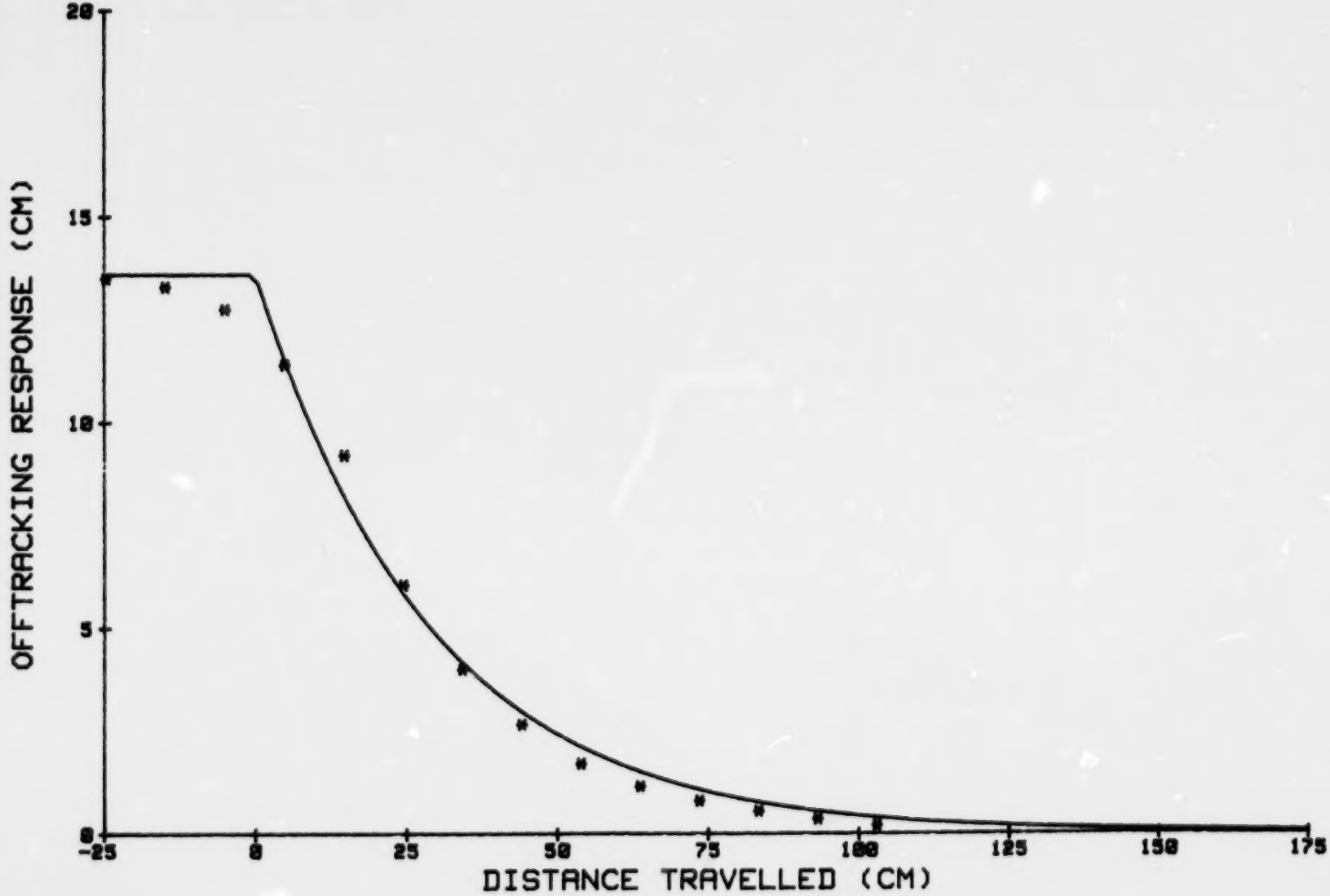

FIG. 9: MODEL TEST – DOUBLE UNIT A-TRAIN EXIT FROM STEADY STATE

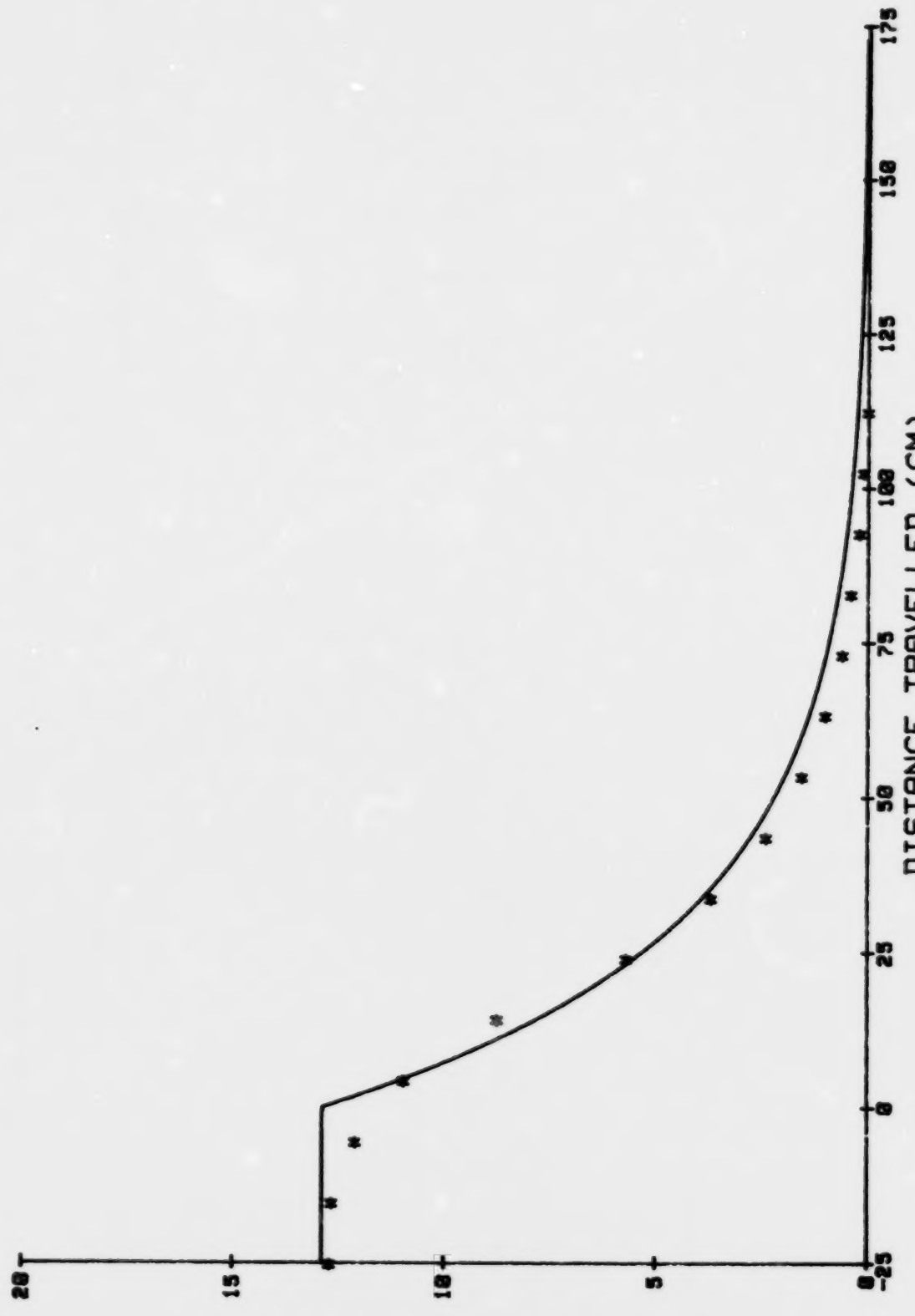

FIG. 10: MODEL TEST – DOUBLE UNIT B-TRAIN EXIT FROM STEADY STATE

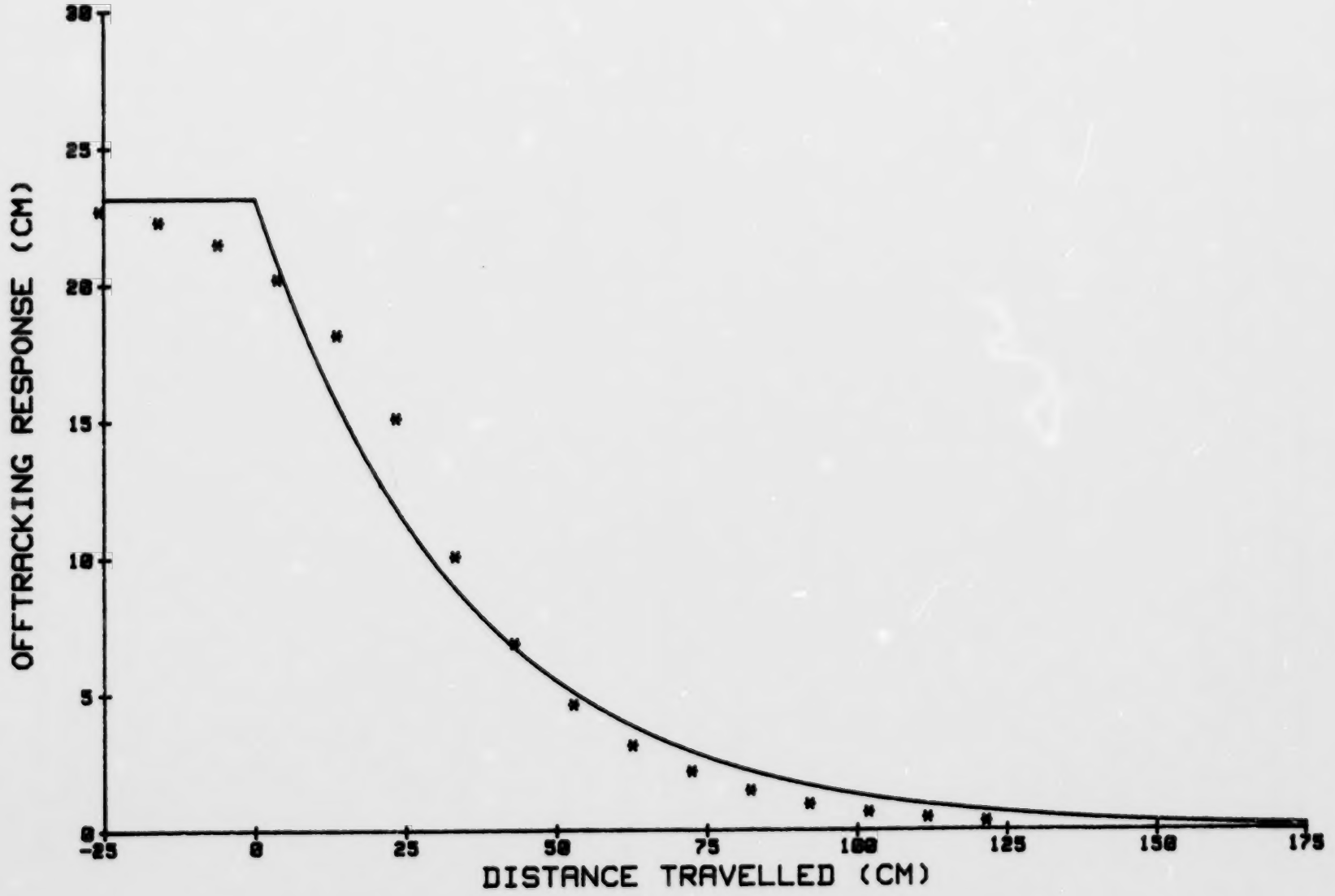

FIG. 11: MODEL TEST – TRIPLE UNIT B-TRAIN EXIT FROM STEADY STATE

In the development of this specific expressions it is convenient to take s = 0 at the rear axle when the lead axle of the vehicle is located at the point where the tangent meets the curve, point G, (Fig. 6). It now follows that, $s \approx wb_e$ when the trailing axle cuts the imaginary radial line OG in Figure 6. As previously mentioned this is the point where Equations (7) and (8) must be equal. At this instant, Equation (7) becomes

$$y_{oc} = OT_{OG} = y_o \exp(-1) \tag{13}$$

where y_o is given by Equation (11).

Recalling that by definition, displacement s in the general expression is equal to zero at the curve commencement. It is appropriate at this time to account for this quantity which is conveniently expressed as $R\theta_{max}$, where θ_{max} is the total angular displacement of the curve expressed in radians.

Equation (8) now becomes

$$OT_{OG} = OT_{max} \exp\left(-\left(\frac{R\theta_{max} - c}{wb_e}\right)\right) \tag{14}$$

Solving for c

$$c = +wb_e \ln\left(\frac{OT_{OG}}{OT_{max}}\right) + R\theta_{max} \tag{15}$$

where OT_{OG} is determined by Equation (13).

Graphical representations of the function showing agreement with measured data from vehicle model tests is seen in Figures 8 to 11. Asterisk points are the measured data.

2.2 Curve Entry

Modelling the vehicle during curve entry can be done successfully by representing the curvilinear system on a rectangular co-ordinate. Referring to Figure 12, if the vehicle were positioned

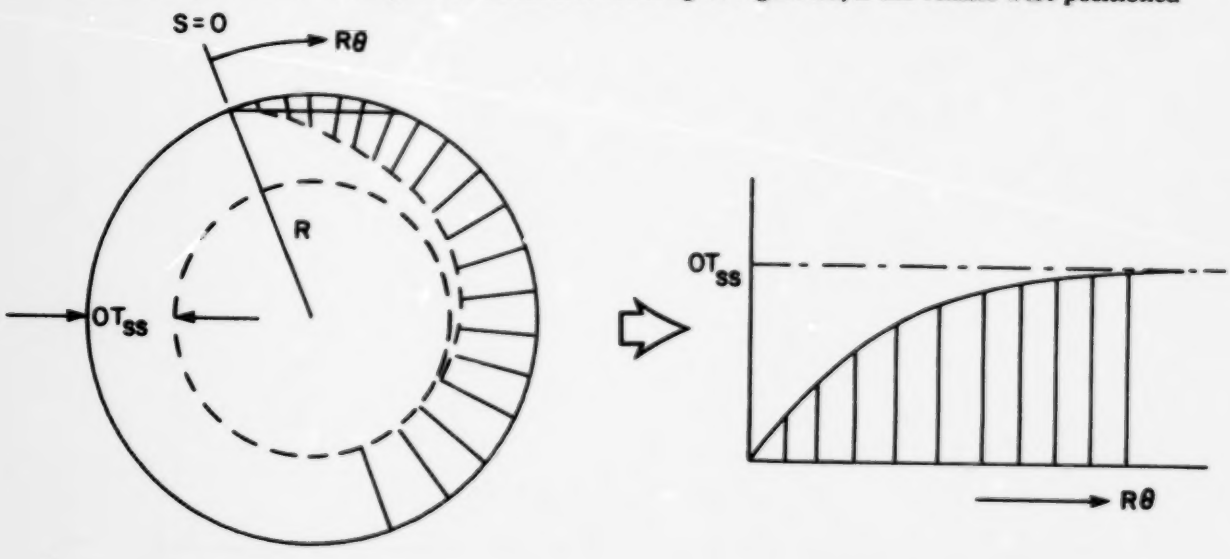

FIG. 12: ILLUSTRATION OF THE SIMPLE TRANSFORMATION FROM POLAR TO THE RECTANGULAR PERSPECTIVE

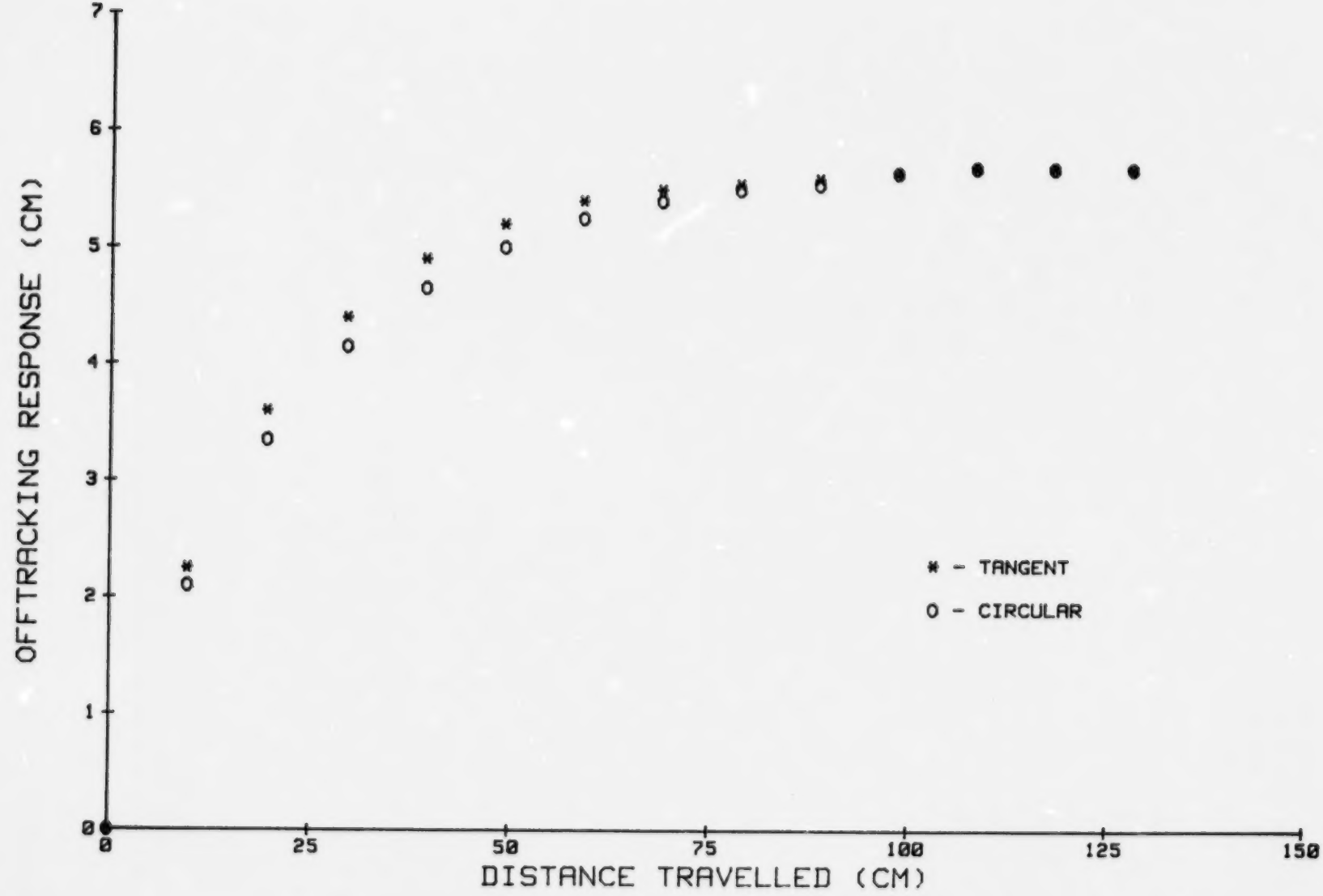

FIG. 13: COMPARISON OF TANGENT AND CONSTANT RADIUS OFF TRACKING

with the "on curve alignment configuration" as defined earlier, with the unit advancing around the constant radius alignment, the rear axle would off track asymptotically to OT_{ss}, the steady state off tracking. The off track transient which occurs through the constant radius alignment is very nearly identical to that which occurs in a tangent alignment with an initial offset equal to OT_{ss} of the curved alignment. The agreement of the measured data is shown in Figure 13.

The ensuring equation takes the form

$$OT = OT_{ss} - OT_{ss} \exp\left(-\frac{s}{wb}\right) \tag{16}$$

To further generalize this relationship to encompass vehicle combinations, the effective wheelbase term is again used.

Equation (16) defines the off tracking from the initial condition of "on curve alignment". For a tangent entry case where the vehicle is initialized with a "tangent alignment" position, one must determine z in Figure 14 so as to shift the exponent of Equation (16) to account for the new "tangent alignment" starting point.

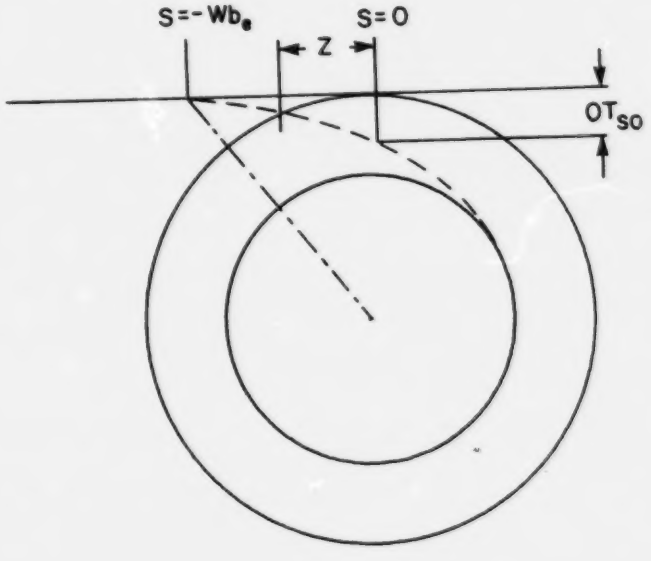

FIG. 14: TANGENT ALIGNMENT CONDITION SHOWING THE OFFSET z

The resulting equation will define off tracking from $-z \leq s \leq c$.

By definition, through the range $-z < s < 0$, the off tracking is measured with respect to the constant radius alignment curve only, therefore, the off tracking will not be with respect to the lead in tangent. In the practical sense, the equation covers the range,

$$0 \leq s \leq c$$

where c is defined in Equation (15).

Equation (16) becomes

$$OT_2 = OT_{ss} - OT_{ss} \exp\left(-\left(\frac{s+z}{wb_e}\right)\right)\Bigg|_{s=o}^{s=c} \qquad (17)$$

where

$$z = 0.6\,\frac{wb_e\,wb_T}{wb_e+wb_T}$$

The derivation of z can be found in Appendix E. A relationship for calculating off tracking with respect to the tangent i.e., $-wb_e < s < o$, can be found by using the following argument. Since the off tracking between $-wb \leqslant s \leqslant o$ is undefined with respect to the tangent, a good approximation can be achieved with an increasing exponential function fulfilling the following condition.

at $s = -wb_e$ $\quad OT = 0$

$s = 0$ $\quad OT = OT_{so}$

OT_{so} (see Fig. 14) is given by Equation (17) which follows

$$OT_{so} = OT_{ss} - OT_{ss} \exp\left(-\frac{z}{wb_e}\right)$$

where OT_{ss} is steady stare off tracking.

The exponential function will have the form

$$OT_1 = OT_{so} - \left(A - A \exp\frac{s}{wb_e}\right)$$

To satisfy the conditions

Condition 1

At $s = -wb_e$, $OT = 0$

since

$$0 = OT_{so} - (A - A\exp^{-1})$$

$$OT_{so} = A(1-\exp^{-1})$$

$\therefore$ the equation becomes

$$OT_1 = OT_{so} - \left(\frac{OT_{so}}{1-\exp^{-1}} - \frac{OT_{so}}{1-\exp^{-1}} \exp\frac{s}{wb_e}\right)$$

$$OT_1 = OT_{so} - \left(\frac{OT_{so}}{.632} - \frac{OT_{so}}{.632} \exp\frac{s}{wb_e}\right)$$

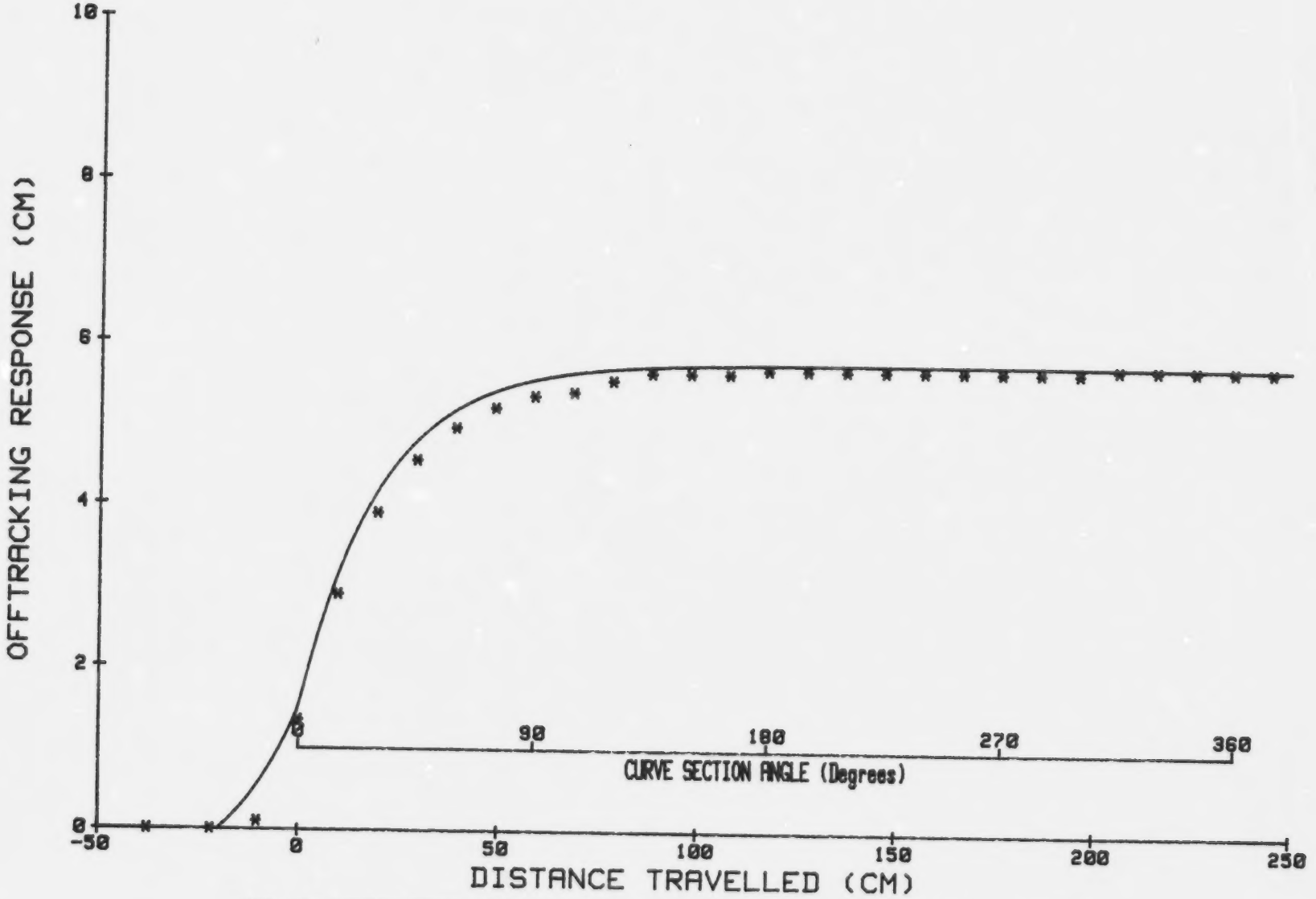

FIG. 15: MODEL TEST – SINGLE UNIT TANGENT TO STEADY STATE 37.5 cm RADIUS

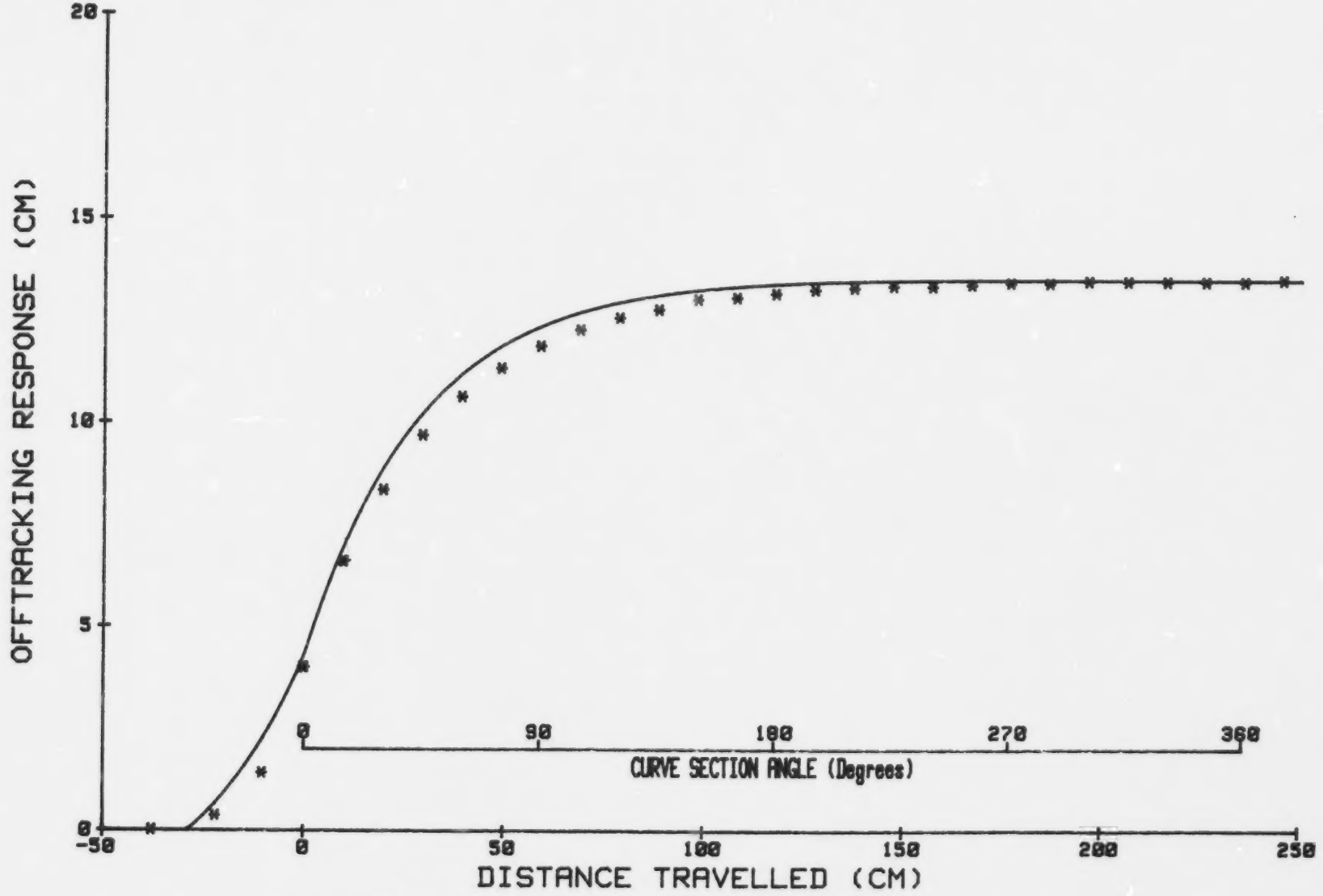

FIG. 16: MODEL TEST – DOUBLE UNIT A-TRAIN TANGENT TO STEADY STATE

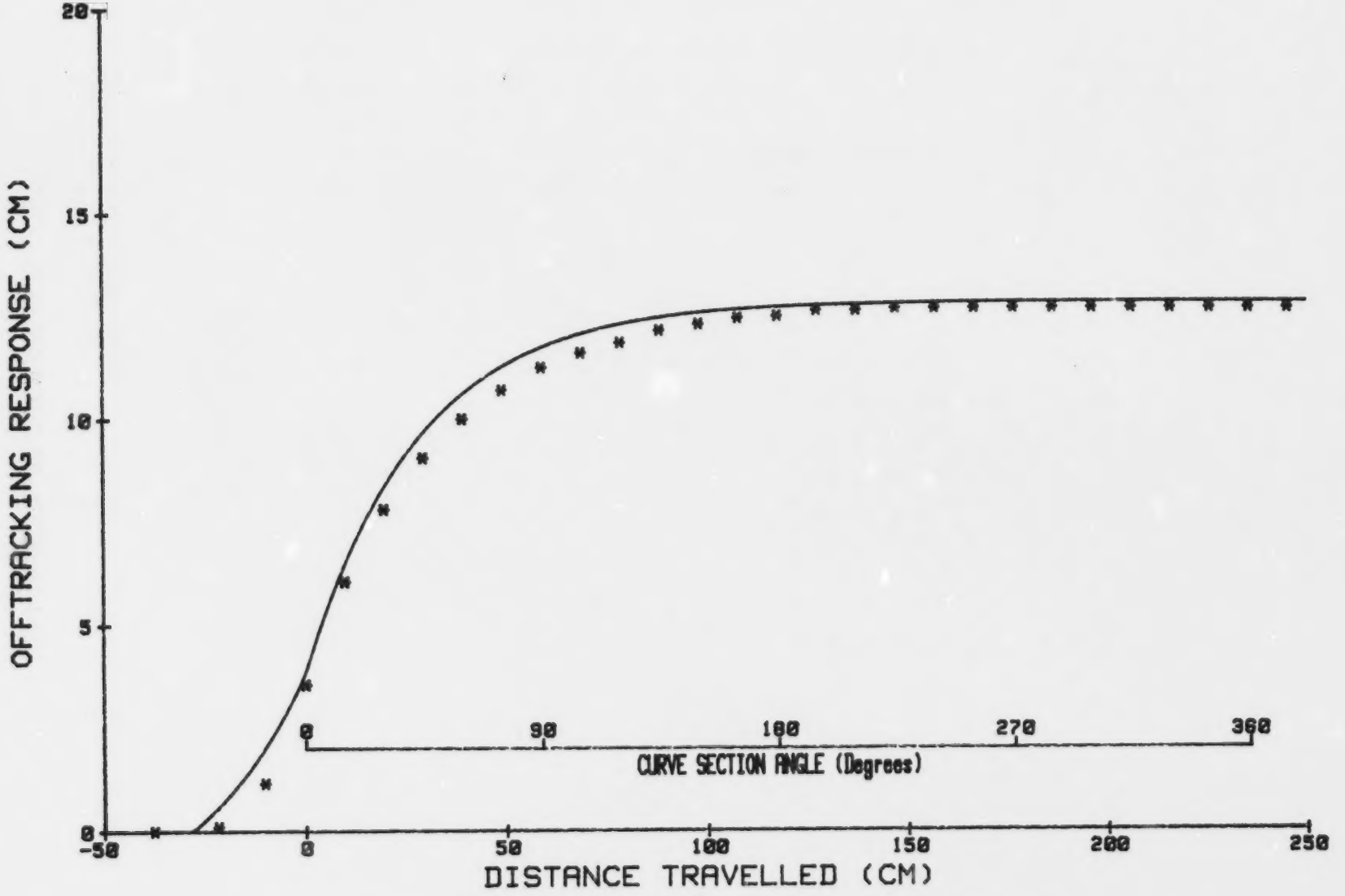

FIG. 17: MODEL TEST – DOUBLE UNIT B-TRAIN TANGENT TO STEADY STATE

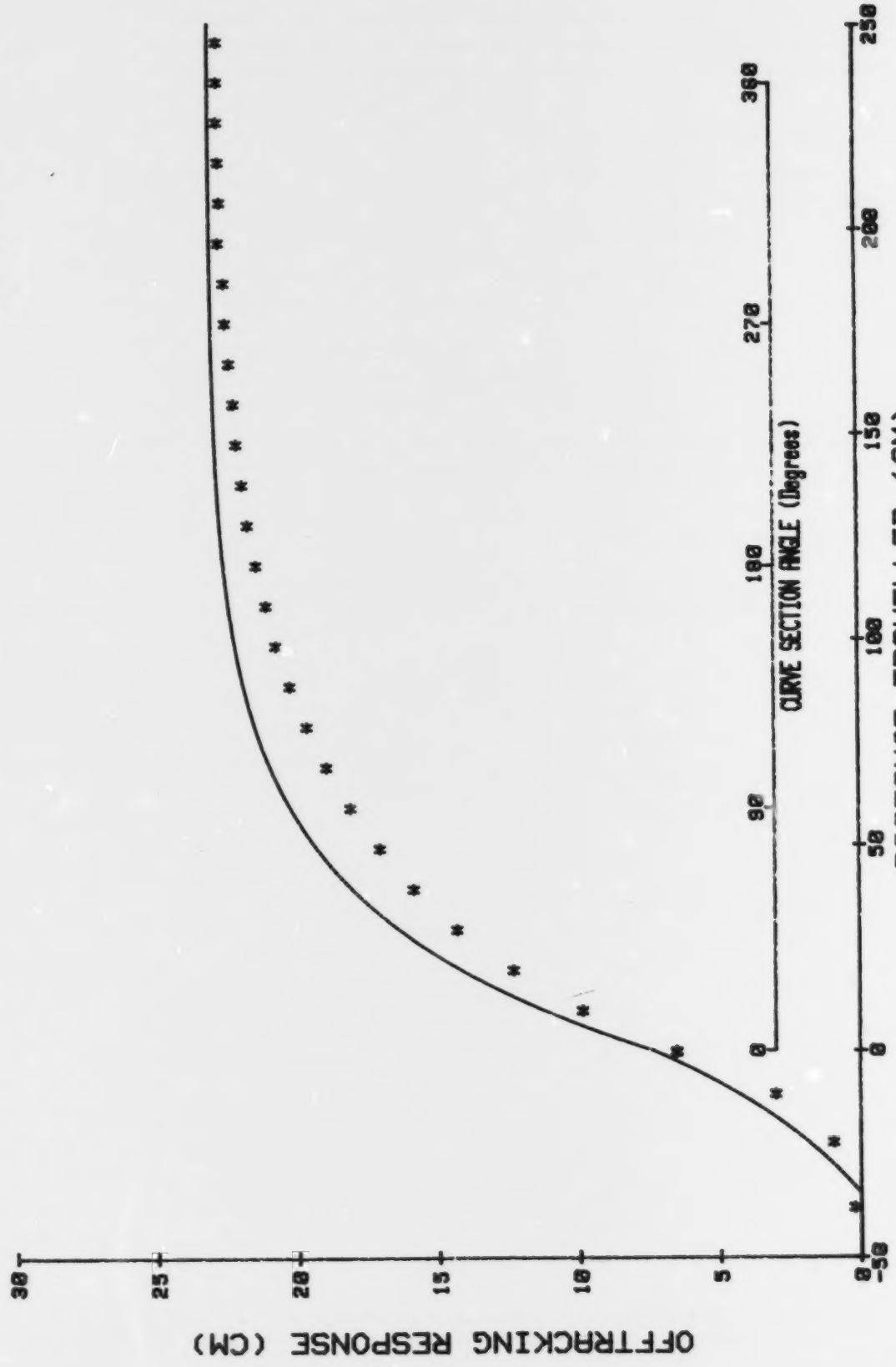

FIG. 18: MODEL TEST – TRIPLE UNIT B-TRAIN TANGENT TO STEADY STATE

or

$$OT_1 = -0.582\,OT_{so} + 1.582\,OT_{so}\,\exp\left(\frac{s}{wb_e}\right)\Bigg|_{s=-wb_e}^{s=0} \tag{18}$$

The generalized curve entry off tracking equations for multiple unit vehicles initially at tangent alignment become

$$OT_1 = -0.582\,OT_{so} + 1.582\,OT_{so}\,\exp\frac{s}{wb_e}\Bigg|_{s=-wb_e}^{s=0} \tag{18}$$

$$OT_2 = OT_{ss} - OT_{ss}\,\exp\left(-\left(\frac{s+z}{wb_e}\right)\right)\Bigg|_{s=0}^{s=c} \tag{17}$$

In Figures 15 to 18 data from physical model experiments is compared to data generated from Equations (17) and (18) for various vehicle configurations entering a constant radius curve from a tangent.

2.3 Combining Curve Entry and Exit

Having solved the entry and exit cases separately, the two equations can be applied in a piecewise manner to solve for the off tracking of a vehicle as it negotiates a constant radius curve with

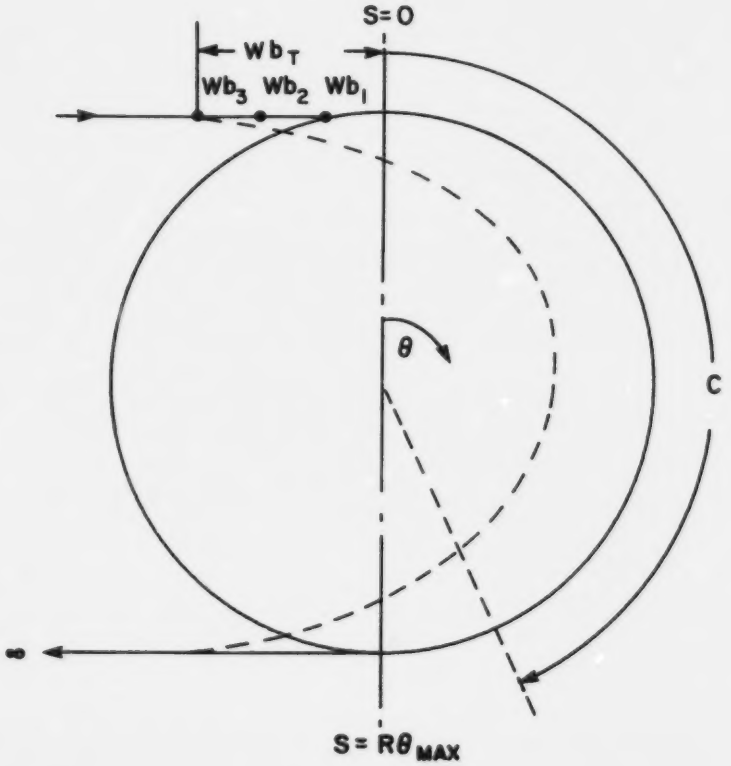

FIG. 19: ILLUSTRATION OF A CURVE WITH A TANGENT LEAD AND EXIT WITH A VEHICLE POSITIONED AT THE CURVE COMMENCEMENT

a tangent lead in and run out. The off tracking results generated are with respect to a track side observer. By this we mean that given a particular alignment path, the off tracking value generated from a value of s, applies to the alignment path at distance s from the start of the curve commencement.

Figure 19 is a general representation of a curve with a tangent lead and exit and a vehicle positioned at the curve commencement. Figure A-2 in Appendix A illustrates where each equation applies.

Equations (18), OT_1 and (17) OT_2, apply from the initial tangent alignment position to point c which is where the unit begins to react to the exit tangent. Equation (8), OT_3, applies from point c to infinity.

2.4 Summary of Equations

The intent of this section is to present the equations required to solve for typical transient off tracking problems in a concise practical format. Used together with examples from Appendix A, it will enable the user to proceed with a solution in a direct and confident manner.

The general piecewise solution is as follows:

$$OT_1 = -0.582\, OT_{so} + 1.582\, OT_{so} \exp\left(\frac{s}{wb_e}\right)\Bigg|_{s=-wb_e}^{s=0}$$

$$OT_2 = OT_{ss} - OT_{ss} \exp\left[(-S-Z)/wb_e\right]\Big|_{s=0}^{s=c}$$

$$OT_3 = OT_{max} \exp\left[(-S+C)/wb_e\right]\Big|_{s=c}^{s=\infty}$$

The constants used in the general solution are defined as follows:

$$wb_T = \sum_{i=1}^{n} wb_i + \sum_{i=1}^{n-1} d_i \qquad (A)$$

$$wb_e = \sqrt{\sum_{i=1}^{n} wb_i^2 - \sum_{i=1}^{n-1} d_i^2} \qquad (B)$$

$$Z = 0.6\frac{wb_T\, wb_e}{wb_T + wb_e} \qquad (C)$$

$$OT_{ss} = R - \sqrt{R^2 - \left(\sum_{i=1}^{n} wb_i^2\right) + \sum_{i=1}^{n-1} d_i^2} \qquad (D)$$

$$C = wb_e\, \ell n\left[\left(wb_e^2 + 2ROT_{ss} - OT_{ss}^2\right)\frac{\exp(-1)}{2R\, OT_{ss}}\right] + R\theta_{max} \qquad (E)$$

C is the distance along the curve at which maximum off tracking (OT_{max}) occurs.

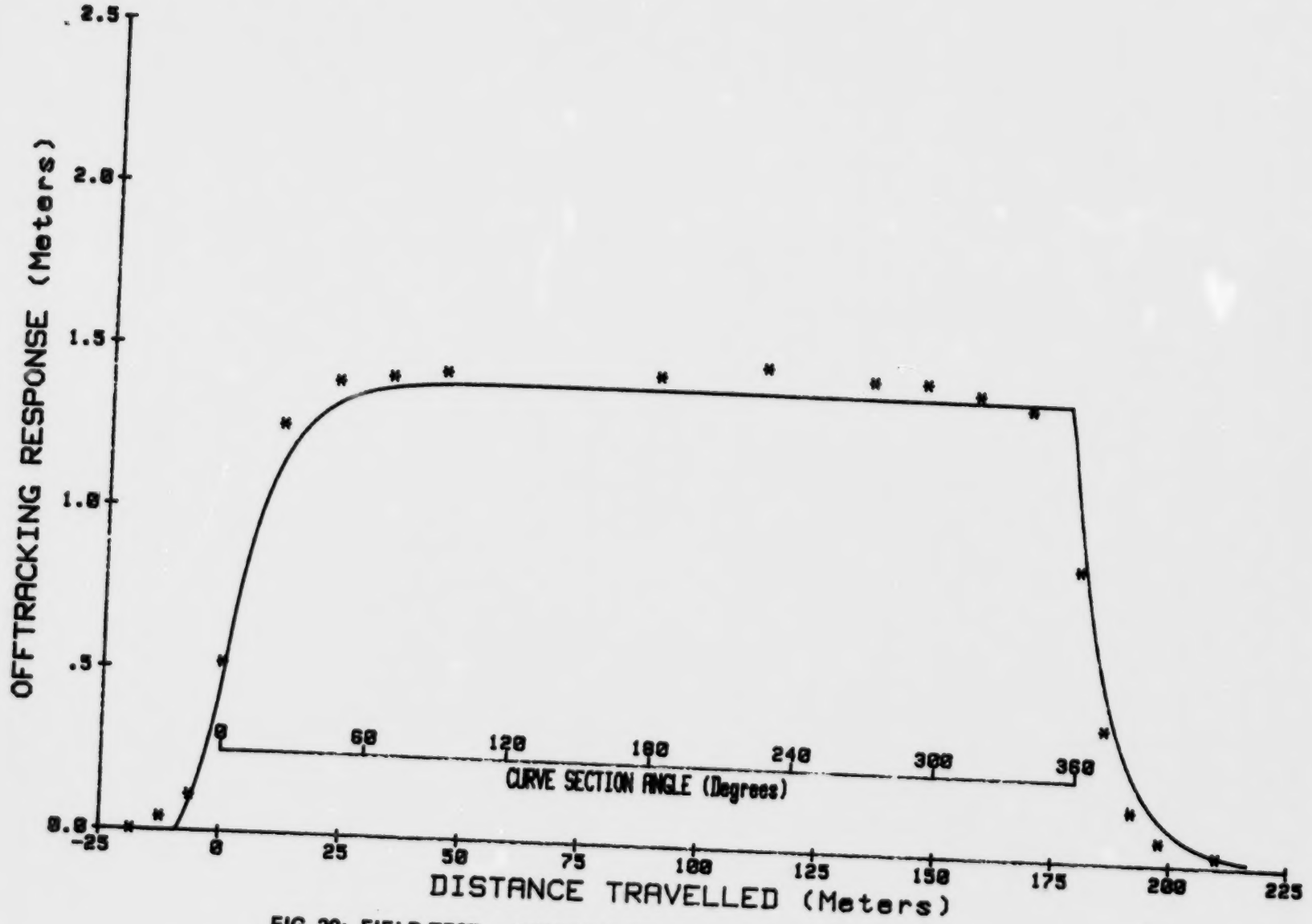

FIG. 20: FIELD TEST – DOUBLE A-TRAIN 28.6 m RADIUS 360 DEG. TURN

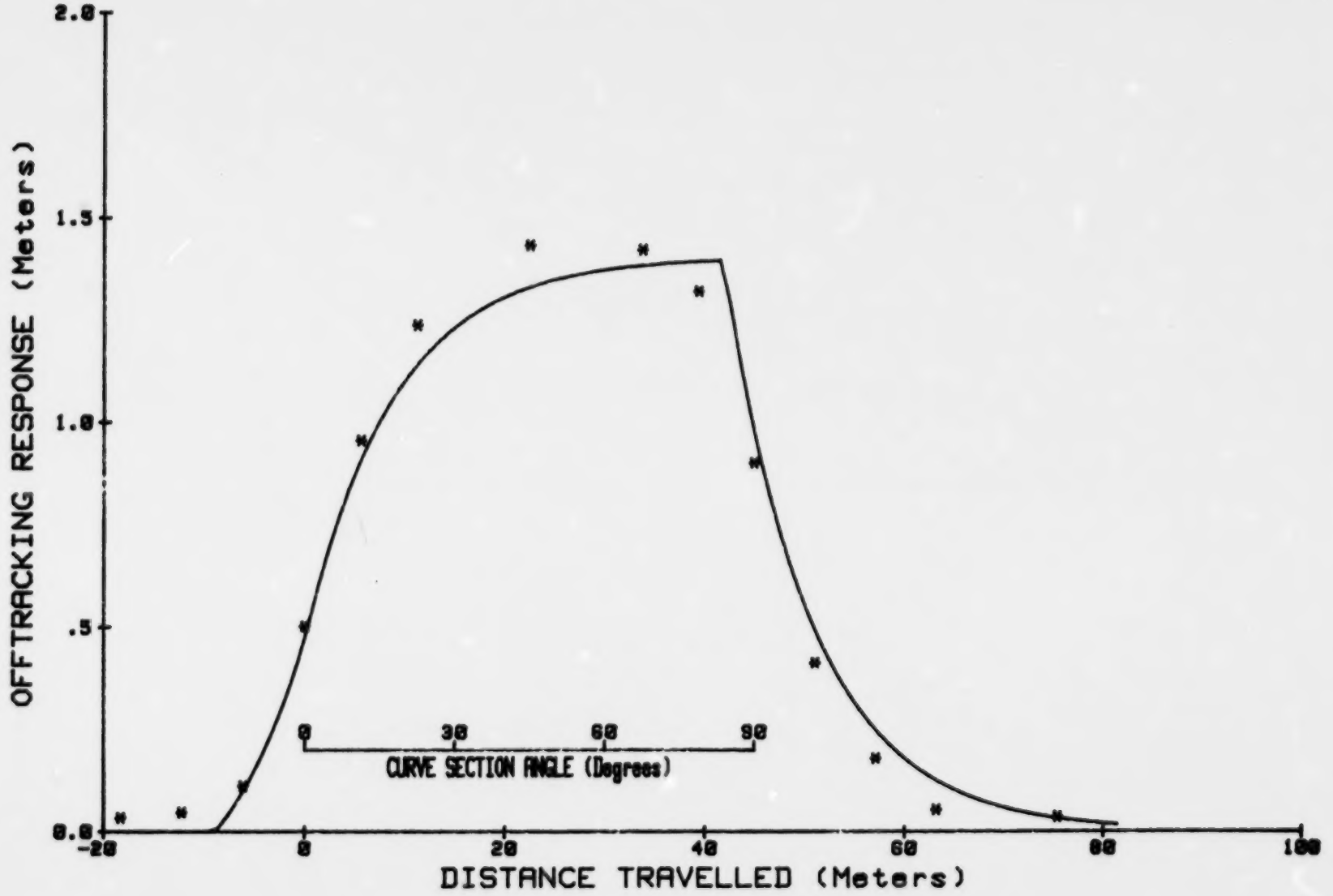

FIG. 21: FIELD TEST – DOUBLE A-TRAIN 28.6 m RADIUS 90 DEG. TURN

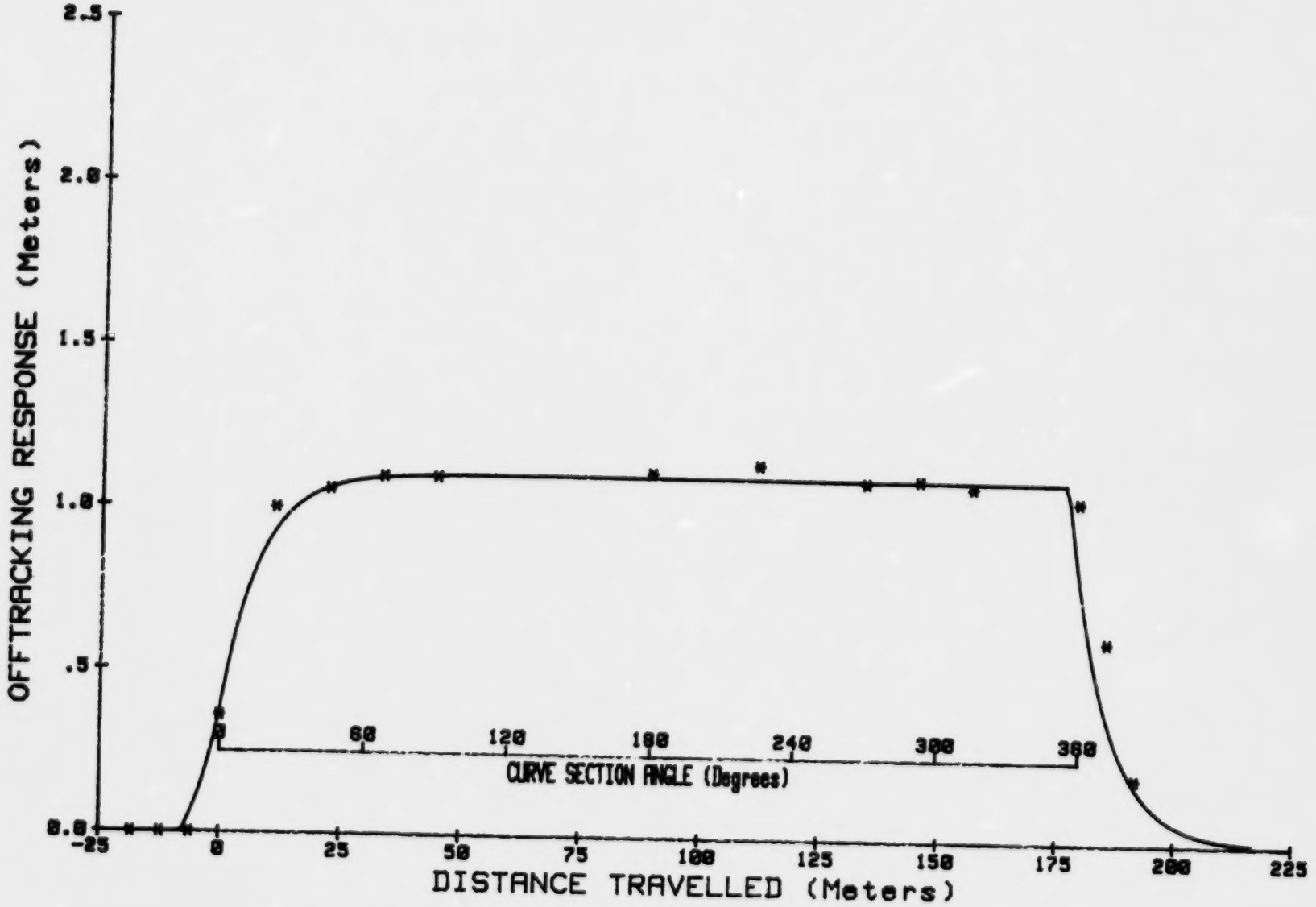

FIG. 22: FIELD TEST – DOUBLE C-TRAIN 28.6 m RADIUS 360 DEG. TURN

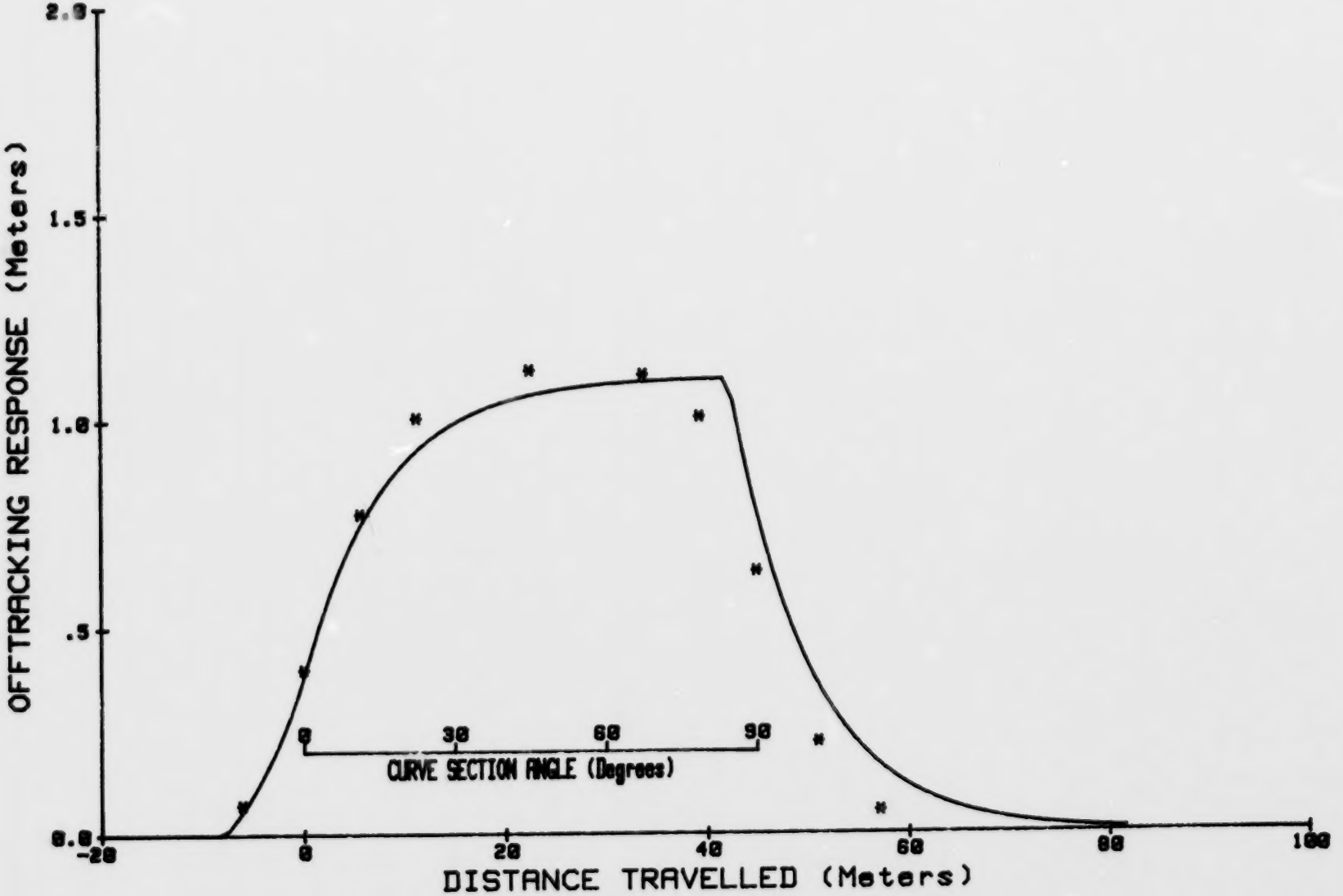

FIG. 23: FIELD TEST – DOUBLE C-TRAIN 28.6 m RADIUS 90 DEG. TURN

$$OT_{max} = OT_{ss} - OT_{ss} \exp\left[(-C-Z)/wb_e\right] \quad (F)$$

$$OT_{so} = OT_{ss} - OT_{ss} \exp\left(\frac{-z}{wb_e}\right) \quad (G)$$

θ_{max} is the total angular sweep of the curve.

The sub equations (A) through (G) are independent of S, the distance travelled through the curve, and therefore remain constant for a specific vehicle in a specific curve.

Rules for equation use and examples problems can be found in Appendix A.

3.0 VALIDATION

Validation was achieved by using both scale models and full scale tractor trailer combinations. The various vehicle configurations tested were multiple units configured as A-, B- and C-trains as well as the simple single unit.

Some of the full scale field test data was supplied pre-processed from an independent source [Ref. 8] and therefore can be considered to be blind test data. Four representative plots containing this blind data together with the mathematical model output are shown in Figures 20 to 23. Excellent agreement between measured and mathematically computed off tracking data was achieved for all full scale tractor trailer combinations considered.

The scale model off tracking data was used in the development of the theory and for broad spectrum validation. Further validation graphs and technical details are presented in Appendix C.

4.0 DISCUSSION AND CONCLUSIONS

The generalized off tracking solution presented here is an analytical solution which allows for a single calculation of off tracking at a specific point along the curve, independent of progression or iteration techniques. For this reason, the solution is field solvable.

The technique permits the calculation of maximum transient off tracking in short duration curves where kinematic steady state conditions are not achieved.

The off tracking results generated using this technique are with respect to a track mounted observer which allows for a convenient comparison of various vehicle combinations relative to any point or points along a curve.

The mathematical model can accurately predict transient off tracking of both single and multiple vehicle combinations through constant radius curves of various angular durations. The solution assumes that the curve has both a tangent lead in and run out. Validation has shown excellent correlation of the solution with both scale model and full size field test results.

The vehicle combination types used for validation are tractor- semi-trainer, A-train double, C-train with a non steering double draw bar dolly (B-dolly) which approximate a B-train, and a C-train with a steering B-dolly axle.

A collection of specific "real world" examples and an outline of the general rules used when applying the equations are contained in Appendix A.

5.0 ACKNOWLEDGEMENTS

The development and validation of this mathematical model required the efforts of a number of people and organizations. The authors wish to thank the Vehicle Systems Group of the Ministry of Transportation and Communications of Ontario and the Canadian Department of National Defence for their co-operation in providing validation test data.

The assistance of the following individuals was greatly appreciated: R.V. Dukkipati, A.M. Garlicki, and C.M. Zwarts for assistance in mathematical verification; E.F. Feser for organizing the semi-trailer field test; J.A. Hutchins for drawing of illustrations; P.E. Klauser for help in reviewing the mathematical analysis and participation in the validation test program; D.D. Marion for the design and fabrication of the scale models; R.J. Senn and J.E. Moore for software development and generation of computer plots.

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APPENDIX A

APPLICATION TECHNIQUES AND EXAMPLE PROBLEMS

GENERAL RULES FOR EQUATION USE

1. Vehicle Dimensions

The vehicle dimensions required for the solution are the wheel base of each unit, and the location of all articulation points. Since the model assumes single axle vehicles, tandem and tri-axle bogies are represented with an imaginary single axle located at the geometric centre of the bogie. In cases where the axle spacing of the bogie is unsymmetrical, such as when the B-dolly is used, the wheel base is taken as the moment centre of the axle group assuming equal forces at all axles.

Referring to the unsymmetrical trailer bogie illustrated in Figure A-1 and taking moments about F_A

$$0 = F_L\, x - F_B\, d_1 - F_C(d_1 + d_2)$$

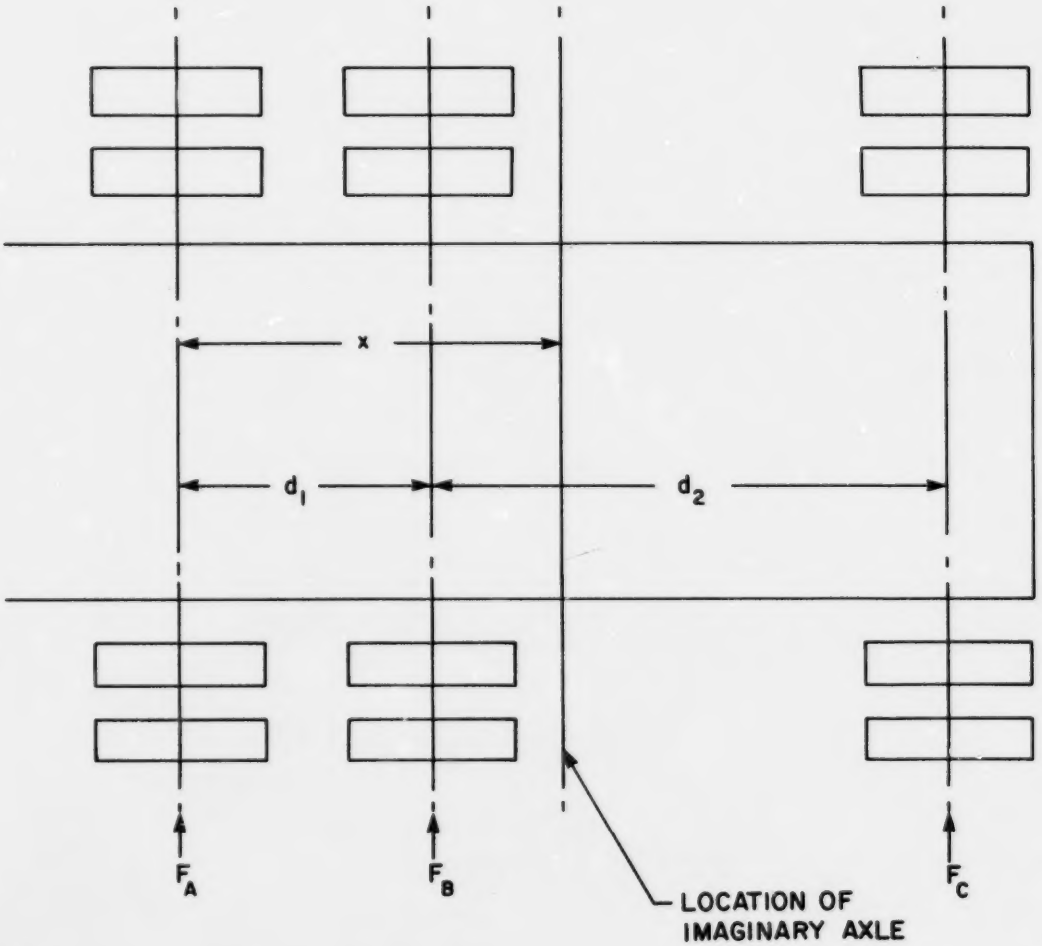


FIG. A-1: ILLUSTRATION OF UNSYMMETRICAL BOGIE

Assuming that tire side forces are equal, i.e.

$$x = 1/3\,(2\,d_1 + d_2) \tag{1-A}$$

Overhangs, fifth wheel settings and hitch points are defined with respect to the imaginary axle.

2. Curve Specification

The radius of curvature is measured to the centre of the tractor steering axle (the centre of the lane on which the tractor is travelling). The curve duration is expressed in radians. The distance term S, is zero at the curve commencement. Distance along the inlet tangent is considered negative and from the curve commencement through the curve and along the exit tangent is positive.

3. Uses of Model

The model can be used in various different ways. For example, if the continuous off tracking response of a vehicle is required, then off tracking is calculated at discrete intervals along the curve and the results are plotted. If only peak off tracking values are required, then Equations (E) and (F) are used. Equation (E) yields the location on the curve where maximum off tracking occurs and (F) determines the off tracking at that point. If the off tracking of a vehicle is required at a specific location through the curve then the distance S at that point is used as the variable in the principal equations.

From a road design perspective, the solution can be used to determine, for example, the location and general size of reinforced road shoulders to accommodate the off tracking of worst case vehicle configurations. This would reduce road costs by eliminating the need for a wide lane throughout a curve.

As shown previously in Section 4.0, the equation required for the calculation of off tracking throughout a curve are as follows.

$$OT_1 = -0.582\,OT_{so} + 1.582\,OT_{so} \exp\left(\frac{s}{wb_e}\right)\Bigg|_{s=-wb_e}^{s=0}$$

$$OT_2 = OT_{ss} - OT_{ss} \exp\left[(-S-Z)/wb_e\right]\Big|_{s=0}^{s=c}$$

$$OT_3 = OT_{max} \exp\left[(-S+C)/wb_e\right]\Big|_{s=c}^{s=\infty}$$

The constants used in the above principal equations are defined as follows:

$$wb_T = \sum_{i=1}^{n} wb_i + \sum_{i=1}^{n-1} d_i \tag{A}$$

$$wb_e = \sqrt{\sum_{i=1}^{n} wb_i^2 - \sum_{i=1}^{n-1} d_i^2} \tag{B}$$

$$Z = 0.6\,\frac{wb_T\,wb_e}{wb_T + wb_e} \tag{C}$$

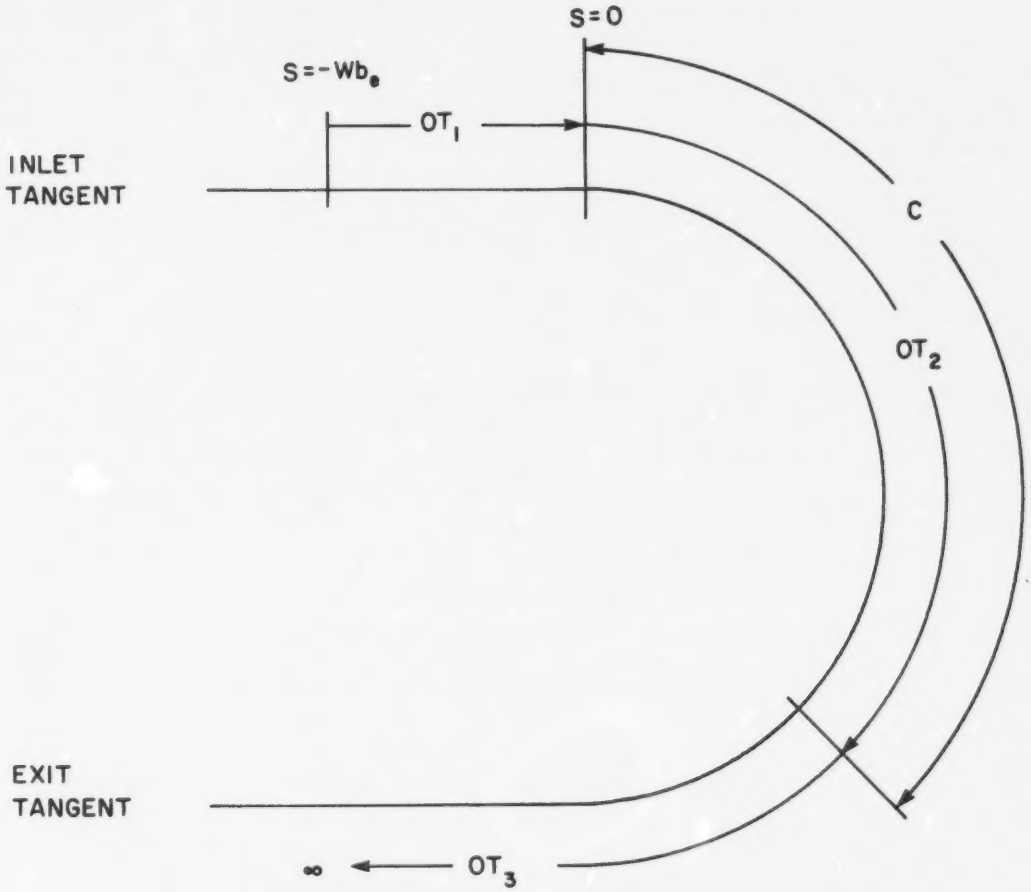


FIG. A-2: CURVE LAYOUT ILLUSTRATING WHERE EACH OF THE THREE MAIN EQUATIONS APPLY

$$OT_{ss} = R - \sqrt{R^2 - \left(\sum_{i=1}^{n} wb_i^2\right) + \sum_{i=1}^{n-1} d_i^2} \qquad (D)$$

$$C = wb_e \, \ell n \left[\left(wb_e^2 + 2ROT_{ss} - OT_{ss}^2\right) \frac{\exp(-1)}{2R\, OT_{ss}}\right] + R\theta_{max} \qquad (E)$$

$$OT_{max} = OT_{ss} - OT_{ss} \exp\left[(-C-Z)/wb_e\right] \qquad (F)$$

$$OT_{so} = OT_{ss} - OT_{ss} \exp\left(\frac{-z}{wb_e}\right) \qquad (G)$$

NOTE: The term C in the point along the curve where the peak transient off tracking takes place.

If a complete off tracking prediction through the curve is required, then all the above equations must be solved.

If only a specific point or portion of the curve is required, then only the equations and constants which apply at that location need be calculated. For additional clarity, Figure A-2 shows where the principal equations apply along the curve.

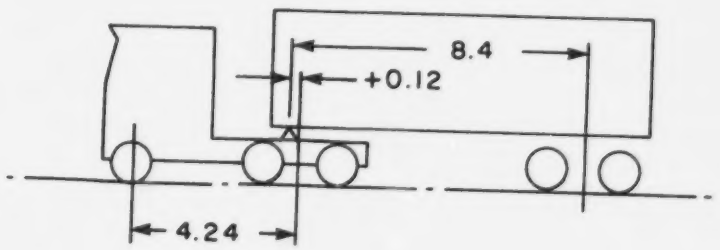
Example Problem

For the benefit of potential users, the following examples reflect a wide variety of vehicle combinations and curve durations. The output of the equation is also presented.

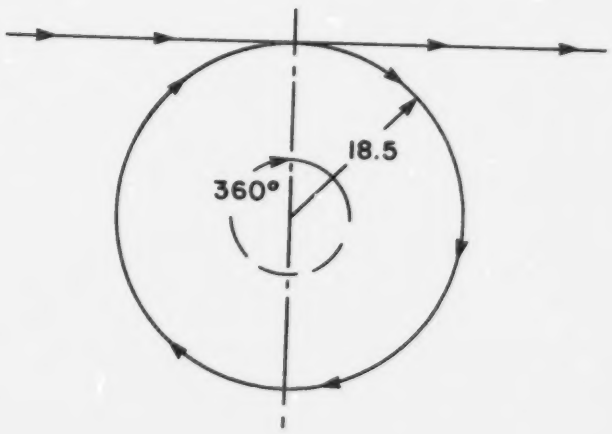
Example 1

Problem: Statement

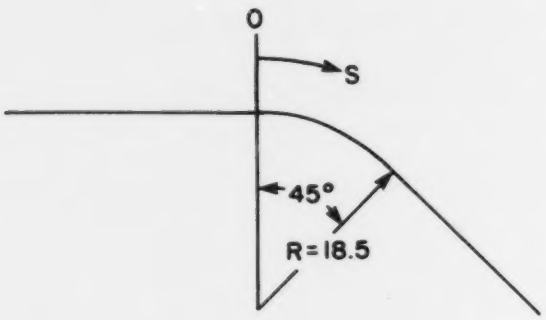
1. Describe the transient off tracking of the illustrated tractor semi-trailer through (A) — 18.5 m constant radius curve of 360° duration with tangent lead in and run out. (B) — 18.5 m constant radius curve of 45° duration with tangent lead in and run out.



UNIT 1



VEHICLE DIMENSIONS AND ROAD ALIGNMENT FOR EXAMPLE 1(A)



ROAD ALIGNMENT EXAMPLE 1(B)

Solution 1(A)

Alignment Specifications

- curve radius (R) 18.5 m at centre of 1st tractor axle
- curve duration 360°; $\theta = 2\pi$ radian
- tangent lead in and run out $\therefore$ the vehicle is assumed not to exhibit any off tracking before curve entry.

Vehicle Specifications

- tractor wheelbase: $(wb_1) = 4.24$ m
- trailer wheelbase: $(wb_2) = 8.40$ m
- tractor fifth wheel setting: $(d_1) = +0.12$ m

$$wb_T = (4.24 + 8.40) - (+0.12) \quad \text{(A)}$$

$$wb_T = 12.52 \text{ m}$$

$$wb_e = \sqrt{(4.24^2 + 8.40^2) - (+0.12^2)} \quad \text{(B)}$$

$$wb_e = 9.41 \text{ m}$$

$$Z = 0.6 \frac{12.52 \times 9.41}{12.52 + 9.41}$$

$$Z = 3.22 \text{ m}$$

$$OT_{ss} = 18.5 - \sqrt{18.5^2 - (4.24^2 + 8.40^2) + (+0.12)^2} \quad \text{(D)}$$

$$OT_{ss} = 2.57 \text{ m}$$

$$C = 9.14\,\ell n\left[(9.41^2 + 2 \times 18.5 \times 2.57 - 2.57^2)\,\frac{\exp(-1)}{2 \times 18.5 \times 2.57}\right] + 18.5 \times 2\pi \qquad \text{(E)}$$

$$C = +112.67 \text{ m}$$

$$OT_{max} = 2.57 - 2.57 \exp\left[(-112.67 - 3.22)/9.41\right] \qquad \text{(F)}$$

$$OT_{max} = 2.57 \text{ m}$$

$$OT_{so} = 2.57 - 2.57 \exp(-3.22/9.41) \qquad OT_{so} = 0.777 \text{ m} \qquad \text{(G)}$$

Having solved for the constants, the three piecewise general off tracking equations become:

$$OT_1 = -0.582\,OT_{so} + 1.582\,OT_{so} \exp(s/9.41)\Big|_{s=-9.41}^{0}$$

$$OT_2 = 2.57 - 2.57 \exp\left[(-S-3.22)/9.41\right]\Big|_{S=0}^{112.67}$$

$$OT_3 = 2.57 \exp\left[(-S + 112.67)/9.41\right]\Big|_{S=112.67}^{-}$$

Problem (1a) TRACTOR-TRAILER 8/12/83

****

ROAD AND VEHICLE SPECIFICATIONS

CURVE DURATION = 360.00 Degrees (6.28 Radians)
CURVE RADIUS = 18.50 M

TRACTOR WHEELBASE= 4.24 M
FIRST TRAILER WHEELBASE= 8.40 M
FIRST DOLLY HITCH LENGTH= 0.00 M
SECOND TRAILER OVERHANG= 0.00 M
THIRD TRAILER WHEELBASE= 0.00 M
THIRD DOLLY HITCH LENGTH= 0.00 M

TRACTOR FIFTH WHEEL POSITION= .12 M
FIRST TRAILER OVERHANG= 0.00 M
SECOND TRAILER WHEELBASE= 0.00 M
SECOND DOLLY HITCH LENGTH= 0.00 M
THIRD TRAILER OVERHANG= 0.00 M
FOURTH TRAILER WHEELBASE 0.00 M

****

STEADY STATE OFF TRACKING = 2.57 M TOTAL WHEEL BASE = 12.52 M
EFFECTIVE WHEEL BASE = 9.41 M

MAXIMUM OFF-TRACKING = 2.57 M occurs 112.67 M at 348.96 Degrees
from the start of the curve.

^^^^^^^^^^

OFFTRACKING = 000	DIST =-12.520	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-10.520	LEAD IN TANGENT
OFFTRACKING = .043	DIST =-8.520	LEAD IN TANGENT EXP FUNC
OFFTRACKING = .156	DIST =-6.520	LEAD IN TANGENT EXP FUNC
OFFTRACKING = .296	DIST =-4.520	LEAD IN TANGENT EXP FUNC
OFFTRACKING = .469	DIST =-2.520	LEAD IN TANGENT EXP FUNC
OFFTRACKING = .682	DIST =-.520	LEAD IN TANGENT EXP FUNC
OFFTRACKING = 1.011	DIST = 1.480	ANG DISPL = 4.584 DEG
OFFTRACKING = 1.310	DIST = 3.480	ANG DISPL = 10.778 DEG
OFFTRACKING = 1.552	DIST = 5.480	ANG DISPL = 16.972 DEG
OFFTRACKING = 1.747	DIST = 7.480	ANG DISPL = 23.166 DEG
OFFTRACKING = 1.905	DIST = 9.480	ANG DISPL = 29.360 DEG
OFFTRACKING = 2.032	DIST = 11.480	ANG DISPL = 35.554 DEG
OFFTRACKING = 2.136	DIST = 13.480	ANG DISPL = 41.748 DEG
OFFTRACKING = 2.219	DIST = 15.480	ANG DISPL = 47.943 DEG
OFFTRACKING = 2.286	DIST = 17.480	ANG DISPL = 54.137 DEG
OFFTRACKING = 2.341	DIST = 19.480	ANG DISPL = 60.331 DEG
OFFTRACKING = 2.385	DIST = 21.480	ANG DISPL = 66.525 DEG
OFFTRACKING = 2.421	DIST = 23.480	ANG DISPL = 72.719 DEG
OFFTRACKING = 2.450	DIST = 25.480	ANG DISPL = 78.913 DEG
OFFTRACKING = 2.473	DIST = 27.480	ANG DISPL = 85.107 DEG
OFFTRACKING = 2.492	DIST = 29.480	ANG DISPL = 91.302 DEG
OFFTRACKING = 2.507	DIST = 31.480	ANG DISPL = 97.496 DEG
OFFTRACKING = 2.519	DIST = 33.480	ANG DISPL = 103.690 DEG
OFFTRACKING = 2.529	DIST = 35.480	ANG DISPL = 109.884 DEG
OFFTRACKING = 2.537	DIST = 37.480	ANG DISPL = 116.078 DEG
OFFTRACKING = 2.544	DIST = 39.480	ANG DISPL = 122.272 DEG
OFFTRACKING = 2.549	DIST = 41.480	ANG DISPL = 128.466 DEG
OFFTRACKING = 2.553	DIST = 43.480	ANG DISPL = 134.661 DEG
OFFTRACKING = 2.557	DIST = 45.480	ANG DISPL = 140.855 DEG
OFFTRACKING = 2.559	DIST = 47.480	ANG DISPL = 147.049 DEG
OFFTRACKING = 2.562	DIST = 49.480	ANG DISPL = 153.243 DEG
OFFTRACKING = 2.564	DIST = 51.480	ANG DISPL = 159.437 DEG
OFFTRACKING = 2.565	DIST = 53.480	ANG DISPL = 165.631 DEG
OFFTRACKING = 2.566	DIST = 55.480	ANG DISPL = 171.825 DEG
OFFTRACKING = 2.567	DIST = 57.480	ANG DISPL = 178.020 DEG
OFFTRACKING = 2.568	DIST = 59.480	ANG DISPL = 184.214 DEG
OFFTRACKING = 2.569	DIST = 61.480	ANG DISPL = 190.408 DEG
OFFTRACKING = 2.569	DIST = 63.480	ANG DISPL = 196.602 DEG
OFFTRACKING = 2.569	DIST = 65.480	ANG DISPL = 202.796 DEG
OFFTRACKING = 2.570	DIST = 67.480	ANG DISPL = 208.990 DEG
OFFTRACKING = 2.570	DIST = 69.480	ANG DISPL = 215.184 DEG
OFFTRACKING = 2.570	DIST = 71.480	ANG DISPL = 221.379 DEG
OFFTRACKING = 2.570	DIST = 73.480	ANG DISPL = 227.573 DEG
OFFTRACKING = 2.571	DIST = 75.480	ANG DISPL = 233.767 DEG
OFFTRACKING = 2.571	DIST = 77.480	ANG DISPL = 239.961 DEG

Problem (1a) TRACTOR-TRAILER 8/12/83

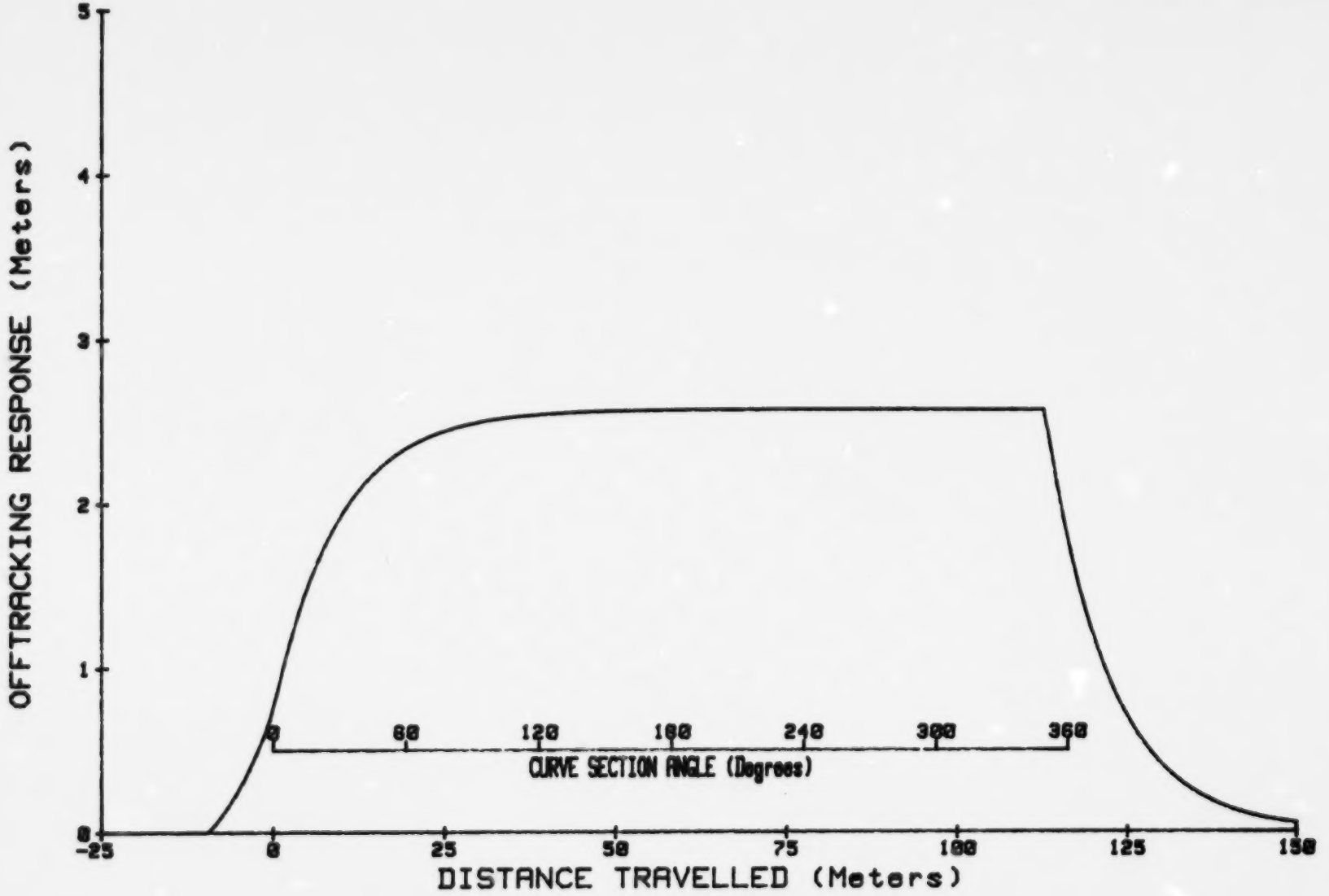
OFFTRACKING	DIST	ANG DISPL
= 2.571	= 79.480	= 246.155 DEG
= 2.571	= 81.480	= 252.349 DEG
= 2.571	= 83.480	= 258.543 DEG
= 2.571	= 85.480	= 264.737 DEG
= 2.571	= 87.480	= 270.932 DEG
= 2.571	= 89.480	= 277.126 DEG
= 2.571	= 91.480	= 283.320 DEG
= 2.571	= 93.480	= 289.514 DEG
= 2.571	= 95.480	= 295.708 DEG
= 2.571	= 97.480	= 301.902 DEG
= 2.571	= 99.480	= 308.096 DEG
= 2.571	= 101.480	= 314.291 DEG
= 2.571	= 103.480	= 320.485 DEG
= 2.571	= 105.480	= 326.679 DEG
= 2.571	= 107.480	= 332.873 DEG
= 2.571	= 109.480	= 339.067 DEG
= 2.571	= 111.480	= 345.261 DEG
= 2.360	= 113.480	= 351.455 DEG
= 1.908	= 115.480	= 357.650 DEG
= 1.543	= 117.480	RUN OUT TANGENT
= 1.247	= 119.480	RUN OUT TANGENT
= 1.008	= 121.480	RUN OUT TANGENT
= .815	= 123.480	RUN OUT TANGENT
= .659	= 125.480	RUN OUT TANGENT
= .533	= 127.480	RUN OUT TANGENT
= .431	= 129.480	RUN OUT TANGENT
= .348	= 131.480	RUN OUT TANGENT
= .282	= 133.480	RUN OUT TANGENT
= .228	= 135.480	RUN OUT TANGENT
= .184	= 137.480	RUN OUT TANGENT
= .149	= 139.480	RUN OUT TANGENT

THE OFFTRACKING VALUES LISTED ARE DEFINED AS THE LATERAL SHIFT OF THE LAST AXLE GROUP OF THE REAR TRAILER WITH RESPECT TO THE PATH FOLLOWED BY THE STEERING AXLE OF THE TRACTOR. ALL POINTS ARE MEASURED FROM THE CENTER LINE OF THE VEHICLE AXIS.

DIST IS THE POINT ON THE PATH WHERE THE GIVEN OFFTRACK VALUE APPLIES. DIST=0 REFERS TO THE START OF THE CURVE. THEREFORE THE LEAD TANGENT PORTION IS NEGATIVE.

ANG DISPL IS THE ANGULAR DISPLACEMENT OF THE CONSTANT RADIUS CURVE WHERE ZERO IS AT THE BEGINNING OF THE CURVE.

********** All units are in METERS **********



GRAPHIC OUTPUT EXAMPLE 1(A) – SEMI TRAILER (18.5 m RADIUS – 360 DEG. TURN)

Solution 1(B)

Alignment Specifications

— same as 1(A) <u>except</u> — curve duration 45° $\theta = \pi/4$ radians

Vehicle Specifications

wb_T, wb_e, z, OT_{ss}, and OT_{so} same as in 1(A)

$$wb_T = 12.52 \text{ m} \quad \text{(A)}$$

$$wb_e = 9.41 \text{ m} \quad \text{(B)}$$

$$Z = 3.22 \text{ m} \quad \text{(C)}$$

$$OT_{ss} = 2.57 \text{ m} \quad \text{(D)}$$

$$OT_{so} = 0.777$$

$$C = 9.41 \ln \left[(9.41^2 + 2 \times 18.5 \times 2.57 - 2.57^2) \frac{\exp(-1)}{2 \times 18.5 \times 2.57} \right] + 18.5 \times \pi/4$$

$$C = +10.97 \text{ m} \quad \text{(E)}$$

$$OT_{max} = 2.57 - 2.57 \exp [(-10.97 - 3.22)/9.41] \quad \text{(F)}$$

$$OT_{max} = 2.00 \text{ m}$$

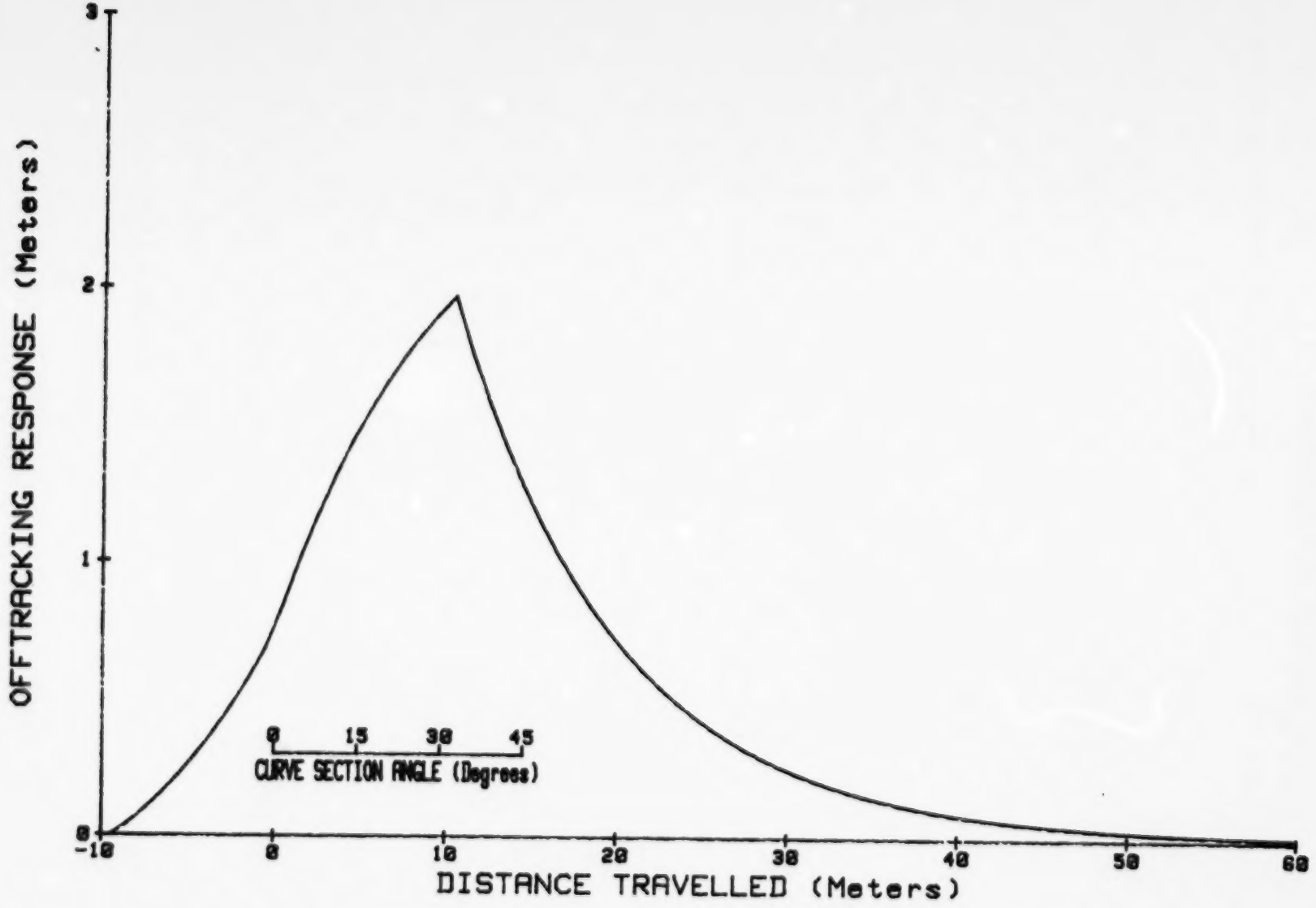
The general off tracking equations become:

$$OT_1 = -0.582 (0.777) + 1.582 (0.777) \exp (s/9.41) \Big|_{s=-9.41}^{0}$$

$$OT_2 = 2.57 - 2.57 \exp [(-S-3.22)/9.41] \Big|_{S=-0}^{10.97}$$

$$OT_3 = 1.87 \exp [(-S + 10.97)/9.41] \Big|_{S=10.97}^{-}$$

```
Problem (1h)    TRACTOR-TRAILER    8/12/83

**********************************************************************

          ROAD AND VEHICLE SPECIFICATIONS

CURVE DURATION = 45.00  Degrees ( .79  Radians)
CURVE RADIUS = 18.50 M

TRACTOR WHEELBASE= 4.24 M          TRACTOR FIFTH WHEEL POSITION= .12 M
FIRST TRAILER WHEELBASE= 8.40 M    FIRST TRAILER OVERHANG= 0.00 M
FIRST DOLLY HITCH LENGTH= 0.00 M   SECOND TRAILER WHEELBASE= 0.00 M
SECOND TRAILER OVERHANG= 0.00 M    SECOND DOLLY HITCH LENGTH= 0.00 M
THIRD TRAILER WHEELBASE= 0.00 M    THIRD TRAILER OVERHANG= 0.00 M
THIRD DOLLY HITCH LENGTH= 0.00 M   FOURTH TRAILER WHEELBASE 0.00 M

**********************************************************************

STEADY STATE OFF TRACKING = 2.57 M  TOTAL WHEEL BASE = 12.52 M
EFFECTIVE WHEEL BASE = 9.41 M

MAXIMUM OFF-TRACKING = 2.00 M occurs  10.97 M at 33.96 Degrees
from the start of the curve.

^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^

 OFFTRACKING = 000       DIST =-12.520    LEAD IN TANGENT
 OFFTRACKING = 000       DIST =-10.520    LEAD IN TANGENT
 OFFTRACKING = .043      DIST =-8.520     LEAD IN TANGENT EXP FUNC
 OFFTRACKING = .156      DIST =-6.520     LEAD IN TANGENT EXP FUNC
 OFFTRACKING = .296      DIST =-4.520     LEAD IN TANGENT EXP FUNC
 OFFTRACKING = .469      DIST =-2.520     LEAD IN TANGENT EXP FUNC
 OFFTRACKING = .682      DIST =-.520      LEAD IN TANGENT EXP FUNC
 OFFTRACKING = 1.011     DIST = 1.480     ANG DISPL = 4.584 DEG
 OFFTRACKING = 1.310     DIST = 3.480     ANG DISPL = 10.778 DEG
 OFFTRACKING = 1.552     DIST = 5.480     ANG DISPL = 16.972 DEG
 OFFTRACKING = 1.747     DIST = 7.480     ANG DISPL = 23.166 DEG
 OFFTRACKING = 1.905     DIST = 9.480     ANG DISPL = 29.360 DEG
 OFFTRACKING = 1.895     DIST = 11.480    ANG DISPL = 35.554 DEG
 OFFTRACKING = 1.532     DIST = 13.480    ANG DISPL = 41.748 DEG
 OFFTRACKING = 1.239     DIST = 15.480    RUN OUT TANGENT
 OFFTRACKING = 1.002     DIST = 17.480    RUN OUT TANGENT
 OFFTRACKING = .810      DIST = 19.480    RUN OUT TANGENT
 OFFTRACKING = .655      DIST = 21.480    RUN OUT TANGENT
 OFFTRACKING = .529      DIST = 23.480    RUN OUT TANGENT
 OFFTRACKING = .428      DIST = 25.480    RUN OUT TANGENT
 OFFTRACKING = .346      DIST = 27.480    RUN OUT TANGENT
 OFFTRACKING = .280      DIST = 29.480    RUN OUT TANGENT
 OFFTRACKING = .226      DIST = 31.480    RUN OUT TANGENT
 OFFTRACKING = .183      DIST = 33.480    RUN OUT TANGENT
 OFFTRACKING = .148      DIST = 35.480    RUN OUT TANGENT
 OFFTRACKING = .120      DIST = 37.480    RUN OUT TANGENT
 OFFTRACKING = .097      DIST = 39.480    RUN OUT TANGENT

 THE OFFTRACKING VALUES LISTED ARE DEFINED AS THE LATERAL
 SHIFT OF THE LAST AXLE GROUP OF THE REAR TRAILER WITH
 RESPECT TO THE PATH FOLLOWED BY THE STEERING AXLE OF THE
 TRACTOR. ALL POINTS ARE MEASURED FROM THE CENTER LINE OF
 THE VEHICLE AXIS.

 DIST IS THE POINT ON THE PATH WHERE THE GIVEN OFFTRACK VALUE
 APPLIES. DIST=0 REFERS TO THE START OF THE CURVE. THEREFORE
 THE LEAD TANGENT PORTION IS NEGATIVE.

 ANG DISPL IS THE ANGULAR DISPLACEMENT OF THE CONSTANT RADIUS
 CURVE WHERE ZERO IS AT THE BEGINNING OF THE CURVE.

     **********  All units are in METERS  **********
```

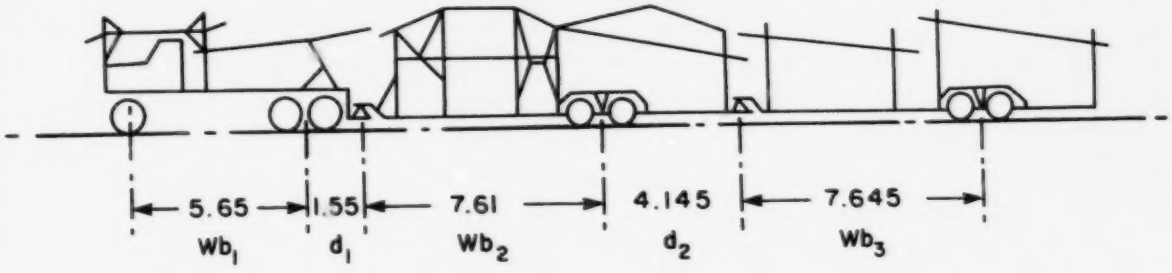


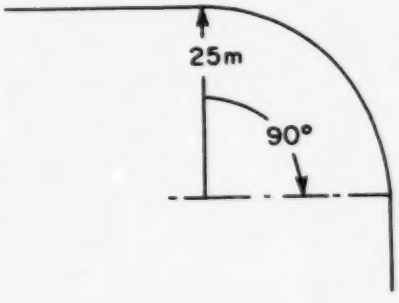
GRAPHIC OUTPUT EXAMPLE 1(B) – SEMI TRAILER (18.5 m RADIUS – 45 DEG. TURN)

Example 2

Problem: Statement

Describe the transient off tracking of the illustrated double car carrier through a 25 m constant radius curve of 90° duration with tangent lead in and run out.





VEHICLE DIMENSIONS AND ROAD ALIGNMENT FOR EXAMPLE 2

Solution

Alignment Specifications

- curve radius (R) 25 m at centre of 1st tractor axle
- curve duration 90° $\theta = \pi/2$ radians
- tangent lead in and run out $\therefore$ the vehicle is assumed not to exhibit any off tracking before curve entry

Vehicle Specifications

- tractor wheelbase: $(wb_1) = 5.65$ m
- tractor 5th sheel position (in this case stinger dimensions. $(d_1) = -1.55$ m

- 1st trailer wheelbase (wb_2) = 7.61 m
- 1st trailer rear overhang (d_2) = 4.15 m
- 2nd trailer wheelbase (wb_3) = 7.65 m

EQUATION

$$wb_T = (5.65 + 1.55 + 7.61 + 4.15 + 7.65) \quad \text{(A)}$$

$$wb_T = 26.61 \text{ m}$$

$$wb_e = \sqrt{5.65^2 + 7.61^2 + 7.65^2 - (-1.55^2 - 4.15^2)} \quad \text{(B)}$$

$$wb_e = 11.35 \text{ m}$$

$$Z = 0.6 \left(\frac{26.61 \times 11.35}{26.61 + 11.35} \right) \quad \text{(C)}$$

$$Z = 4.77 \text{ m}$$

$$OT_{ss} = 25 - \sqrt{25^2 - (5.65^2 + 7.61^2 + 7.65^2) + (-1.55^2 + 4.15^2)} \quad \text{(D)}$$

$$OT_{ss} = 2.72 \text{ m}$$

$$C = 11.35 \, \ell n \left[(11.35^2 + 2 \times 25.0 \times 2.72 - 2.72^2) \frac{\exp(-1)}{2 \times 25.0 \times 2.72} \right] + 25 \times \pi/2 \quad \text{(E)}$$

$$C = 35.15$$

$$OT_{max} = 2.72 - 2.72 \exp [(-35.15 - 4.77)/11.35] \quad \text{(F)}$$

$$OT_{max} = 2.64 \text{ m}$$

$$OT_{so} = 2.72 - 2.72 \exp (-4.77/11.35) \quad \text{(G)}$$

$$OT_{so} = 0.933 \text{ m}$$

Having solved for the constants, the three piecewise general off tracking equations become:

$$OT_1 = -0.582 (.933) - 1.582 (.933) \exp (s/11.35) \Big|_{s=-11.35}^{0}$$

$$OT_2 = 2.72 - 2.72 \exp \left((-S - 4.77)/11.35 \right) \Big|_{S=-0}^{35.15}$$

$$OT_3 = 2.39 \exp \left((-S + 35.15)/11.35 \right) \Big|_{S=35.15}^{-}$$

Problem (2) R-TRAIN WITH 2 TRAILERS 8/12/83

****

ROAD AND VEHICLE SPECIFICATIONS

CURVE DURATION = 90.00 Degrees (1.57 Radians)
CURVE RADIUS = 25.00 M

TRACTOR WHEELBASE= 5.65 M
FIRST TRAILER WHEELBASE= 7.61 M
FIRST DOLLY HITCH LENGTH= 0.00 M
SECOND TRAILER OVERHANG= 0.00 M
THIRD TRAILER WHEELBASE= 0.00 M
THIRD DOLLY HITCH LENGTH= 0.00 M

TRACTOR FIFTH WHEEL POSITION=-1.55 M
FIRST TRAILER OVERHANG= 4.15 M
SECOND TRAILER WHEELBASE= 7.65 M
SECOND DOLLY HITCH LENGTH= 0.00 M
THIRD TRAILER OVERHANG= 0.00 M
FOURTH TRAILER WHEELBASE 0.00 M

****

STEADY STATE OFF TRACKING = 2.72 M TOTAL WHEEL BASE = 26.61 M
EFFECTIVE WHEEL BASE = 11.35 M

MAXIMUM OFF-TRACKING = 2.64 M occurs 35.15 M at 80.56 Degrees
from the start of the curve.

^^^^^^^^^^

OFFTRACKING = 000	DIST =-26.610	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-24.610	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-22.610	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-20.610	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-18.610	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-16.610	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-14.610	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-12.610	LEAD IN TANGENT
OFFTRACKING = .036	DIST =-10.610	LEAD IN TANGENT EXP FUNC
OFFTRACKING = .148	DIST =-8.610	LEAD IN TANGENT EXP FUNC
OFFTRACKING = .282	DIST =-6.610	LEAD IN TANGENT EXP FUNC
OFFTRACKING = .441	DIST =-4.610	LEAD IN TANGENT EXP FUNC
OFFTRACKING = .631	DIST =-2.610	LEAD IN TANGENT EXP FUNC
OFFTRACKING = .858	DIST =-.610	LEAD IN TANGENT EXP FUNC
OFFTRACKING = 1.141	DIST = 1.390	ANG DISPL = 3.186 DEG
OFFTRACKING = 1.397	DIST = 3.390	ANG DISPL = 7.769 DEG
OFFTRACKING = 1.611	DIST = 5.390	ANG DISPL = 12.353 DE
OFFTRACKING = 1.791	DIST = 7.390	ANG DISPL = 16.937 DEG
OFFTRACKING = 1.941	DIST = 9.390	ANG DISPL = 21.520 DEG
OFFTRACKING = 2.068	DIST = 11.390	ANG DISPL = 26.104 DEG
OFFTRACKING = 2.174	DIST = 13.390	ANG DISPL = 30.688 DEG
OFFTRACKING = 2.262	DIST = 15.390	ANG DISPL = 35.271 DEG
OFFTRACKING = 2.337	DIST = 17.390	ANG DISPL = 39.855 DEG
OFFTRACKING = 2.399	DIST = 19.390	ANG DISPL = 44.439 DEG
OFFTRACKING = 2.452	DIST = 21.390	ANG DISPL = 49.022 DEG
OFFTRACKING = 2.495	DIST = 23.390	ANG DISPL = 53.606 DEG
OFFTRACKING = 2.532	DIST = 25.390	ANG DISPL = 58.190 DEG
OFFTRACKING = 2.563	DIST = 27.390	ANG DISPL = 62.773 DEG
OFFTRACKING = 2.589	DIST = 29.390	ANG DISPL = 67.357 DEG
OFFTRACKING = 2.611	DIST = 31.390	ANG DISPL = 71.941 DEG
OFFTRACKING = 2.629	DIST = 33.390	ANG DISPL = 76.524 DEG
OFFTRACKING = 2.588	DIST = 35.390	ANG DISPL = 81.108 DEG
OFFTRACKING = 2.169	DIST = 37.390	ANG DISPL = 85.692 DEG
OFFTRACKING = 1.819	DIST = 39.390	RUN OUT TANGENT
OFFTRACKING = 1.525	DIST = 41.390	RUN OUT TANGENT
OFFTRACKING = 1.278	DIST = 43.390	RUN OUT TANGENT
OFFTRACKING = 1.072	DIST = 45.390	RUN OUT TANGENT
OFFTRACKING = .899	DIST = 47.390	RUN OUT TANGENT
OFFTRACKING = .753	DIST = 49.390	RUN OUT TANGENT
OFFTRACKING = .632	DIST = 51.390	RUN OUT TANGENT
OFFTRACKING = .530	DIST = 53.390	RUN OUT TANGENT
OFFTRACKING = .444	DIST = 55.390	RUN OUT TANGENT
OFFTRACKING = .372	DIST = 57.390	RUN OUT TANGENT
OFFTRACKING = .312	DIST = 59.390	RUN OUT TANGENT
OFFTRACKING = .262	DIST = 61.390	RUN OUT TANGENT
OFFTRACKING = .219	DIST = 63.390	RUN OUT TANGENT

Problem (2) B-TRAIN WITH 2 TRAILERS 8/12/83

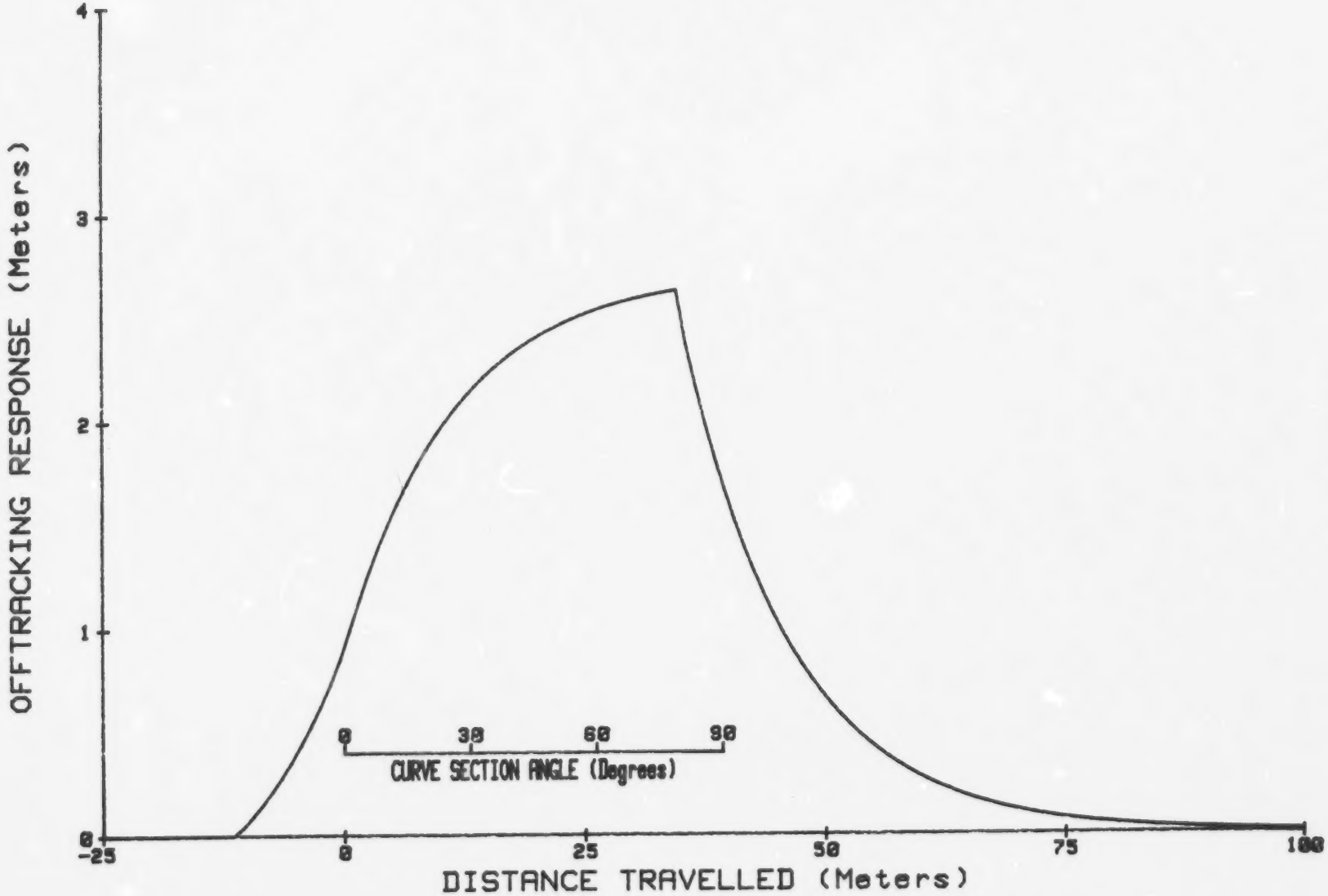
OFFTRACKING = .184	DIST = 65.390	RUN OUT TANGENT
OFFTRACKING = .154	DIST = 67.390	RUN OUT TANGENT
OFFTRACKING = .129	DIST = 69.390	RUN OUT TANGENT
OFFTRACKING = .108	DIST = 71.390	RUN OUT TANGENT
OFFTRACKING = .091	DIST = 73.390	RUN OUT TANGENT
OFFTRACKING = .076	DIST = 75.390	RUN OUT TANGENT
OFFTRACKING = .064	DIST = 77.390	RUN OUT TANGENT
OFFTRACKING = .054	DIST = 79.390	RUN OUT TANGENT
OFFTRACKING = .045	DIST = 81.390	RUN OUT TANGENT
OFFTRACKING = .038	DIST = 83.390	RUN OUT TANGENT
OFFTRACKING = .032	DIST = 85.390	RUN OUT TANGENT
OFFTRACKING = .026	DIST = 87.390	RUN OUT TANGENT
OFFTRACKING = .022	DIST = 89.390	RUN OUT TANGENT
OFFTRACKING = .019	DIST = 91.390	RUN OUT TANGENT

THE OFFTRACKING VALUES LISTED ARE DEFINED AS THE LATERAL SHIFT OF THE LAST AXLE GROUP OF THE REAR TRAILER WITH RESPECT TO THE PATH FOLLOWED BY THE STEERING AXLE OF THE TRACTOR. ALL POINTS ARE MEASURED FROM THE CENTER LINE OF THE VEHICLE AXIS.

DIST IS THE POINT ON THE PATH WHERE THE GIVEN OFFTRACK VALUE APPLIES. DIST=0 REFERS TO THE START OF THE CURVE. THEREFORE THE LEAD TANGENT PORTION IS NEGATIVE.

ANG DISPL IS THE ANGULAR DISPLACEMENT OF THE CONSTANT RADIUS CURVE WHERE ZERO IS AT THE BEGINNING OF THE CURVE.

********** All units are in METERS **********

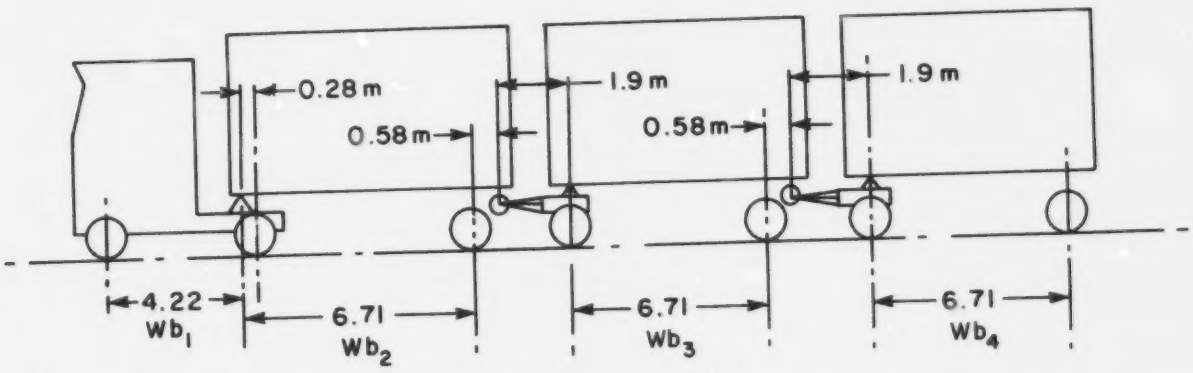


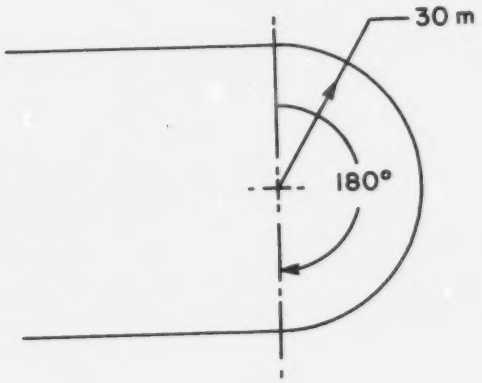
GRAPHIC OUTPUT EXAMPLE 2 – DOUBLE CAR CARRIER (25 m RADIUS – 90 DEG. TURN)

Example 3

Problem: Statement

Describe the transient off tracking of the illustrated triple train through 30 m constant radius curve of 180° duration with tangent lead in and run out.





VEHICLE DIMENSIONS AND ROAD ALIGNMENT EXAMPLE 3

Solution

Alignment Specifications

- curve radius (R) 30 m at centre of 1st tractor axle
- curve duration 180° $\theta = \pi$ radians
- tangent lead in and run out ∴ vehicle assumed not to exhibit any off tracking before curve entry

Vehicle Specifications

- tractor wheelbase (wb_1) = 4.22 m
- tractor 5th wheel position (d_1) = +0.28 m
- 1st trailer wheelbase (wb_2) = 6.71 m
- 1st trailer overhang (d_2) = 0.58 m
- 1st dolly hitch length (wb_3) = 1.90 m
- 2nd trailer wheelbase (wb_4) = 6.71 m
- 2nd trailer overhang (d_3) = 0.58 m
- 2nd dolly hitch length (wb_5) = 1.90 m
- 3rd trailer wheelbase (wb_6) = 6.71 m

EQUATION

$$wb_T = (4.22 - 0.28 + 6.71 + 0.58 + 1.90 + 6.71 + 0.58 + 1.90 + 6.71) \quad \text{(A)}$$

$$wb_T = 29.03 \text{ m}$$

$$wb_e = \sqrt{4.22^2 + 6.71^2 + 1.90^2 + 6.71^2 + 1.90^2 + 6.71^2 - 0.28^2 - 0.58^2 - 0.58^2} \quad \text{(B)}$$

$$wb_e = 12.62 \text{ m}$$

$$Z = 0.6 \left(\frac{29.03 \times 12.62}{29.03 + 12.62} \right) \quad \text{(C)}$$

$$Z = 5.28 \text{ m}$$

$$OT_{ss} = 30 - \sqrt{30^2 - 160.10 + 0.88} \quad \text{(D)}$$

$$OT_{ss} = 2.79 \text{ m}$$

$$C = 12.62 \, \ell n \left[(12.62^2 + 2 \times 30 \times 2.79 - 2.79^2) \frac{\exp(-1)}{2 \times 30 \times 2.79} \right] + 30\pi \quad \text{(E)}$$

$$C = 89.77$$

$$OT_{max} = 2.79 - 2.79 \exp [(-89.77 - 5.28)/12.62] \quad \text{(F)}$$

$$OT_{max} = 2.79 \text{ m}$$

$$OT_{so} = 2.79 - 2.79 \exp (-5.28/12.62) \quad \text{(G)}$$

$$OT_{so} = 0.954 \text{ m}$$

Having solved for the constants, the three piecewise general off tracking equations become:

$$OT_1 = -0.582\,(.954) + 1.582\,(0.954)\exp(3/12.61)\Big|_{s=-12.62}^{0}$$

$$OT_2 = 2.79 - 2.79\exp[(-S-5.28)/12.61]\Big|_{S=0}^{89.77}$$

$$OT_3 = 2.76\exp[(-S+89.77)/12.62]\Big|_{s=89.77}^{-}$$

Problem (3) A-TRAIN WITH 3 TRAILERS 8/12/83

****

ROAD AND VEHICLE SPECIFICATIONS

CURVE DURATION = 180.00 Degrees (3.14 Radians)
CURVE RADIUS = 30.00 M

TRACTOR WHEELBASE= 4.22 M
FIRST TRAILER WHEELBASE= 6.71 M
FIRST DOLLY HITCH LENGTH= 1.90 M
SECOND TRAILER OVERHANG= .58 M
THIRD TRAILER WHEELBASE= 6.71 M
THIRD DOLLY HITCH LENGTH= 0.00 M

TRACTOR FIFTH WHEEL POSITION= .28 M
FIRST TRAILER OVERHANG= .58 M
SECOND TRAILER WHEELBASE= 6.71 M
SECOND DOLLY HITCH LENGTH= 1.90 M
THIRD TRAILER OVERHANG= 0.00 M
FOURTH TRAILER WHEELBASE 0.00 M

****

STEADY STATE OFF TRACKING = 2.79 M TOTAL WHEEL BASE = 29.03 M
EFFECTIVE WHEEL BASE = 12.62 M

MAXIMUM OFF-TRACKING = 2.78 M occurs 89.77 M at 171.46 Degrees
from the start of the curve.

^^^^^^^^^^

OFFTRACKING = 000	DIST =-29.030	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-27.030	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-25.030	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-23.030	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-21.030	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-19.030	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-17.030	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-15.030	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-13.030	LEAD IN TANGEN
OFFTRACKING = .075	DIST =-11.030	LEAD IN TANGENT EXP FUNC
OFFTRACKING = .182	DIST =-9.030	LEAD IN TANGENT EXP FUNC
OFFTRACKING = .309	DIST =-7.030	LEAD IN TANGENT EXP FUNC
OFFTRACKING = .457	DIST =-5.030	LEAD IN TANGENT EXP FUNC
OFFTRACKING = .630	DIST =-3.030	LEAD IN TANGENT EXP FUNC
OFFTRACKING = .834	DIST =-1.030	LEAD IN TANGENT EXP FUNC
OFFTRACKING = 1.087	DIST = .970	ANG DISPL = 1.853 DEG
OFFTRACKING = 1.336	DIST = 2.970	ANG DISPL = 5.672 DEG
OFFTRACKING = 1.548	DIST = 4.970	ANG DISPL = 9.492 DEG
OFFTRACKING = 1.730	DIST = 6.970	ANG DISPL = 13.312 DEG
OFFTRACKING = 1.884	DIST = 8.970	ANG DISPL = 17.131 DEG
OFFTRACKING = 2.016	DIST = 10.970	ANG DISPL = 20.951 DEG
OFFTRACKING = 2.129	DIST = 12.970	ANG DISPL = 24.771 DEG
OFFTRACKING = 2.225	DIST = 14.970	ANG DISPL = 28.591 DEG
OFFTRACKING = 2.307	DIST = 16.970	ANG DISPL = 32.410 DEG
OFFTRACKING = 2.377	DIST = 18.970	ANG DISPL = 36.230 DEG
OFFTRACKING = 2.437	DIST = 20.970	ANG DISPL = 40.050 DEG
OFFTRACKING = 2.488	DIST = 22.970	ANG DISPL = 43.869 DEG
OFFTRACKING = 2.531	DIST = 24.970	ANG DISPL = 47.689 DEG
OFFTRACKING = 2.569	DIST = 26.970	ANG DISPL = 51.509 DEG
OFFTRACKING = 2.600	DIST = 28.970	ANG DISPL = 55.329 DEG
OFFTRACKING = 2.627	DIST = 30.970	ANG DISPL = 59.148 DEG
OFFTRACKING = 2.651	DIST = 32.970	ANG DISPL = 62.968 DEG
OFFTRACKING = 2.670	DIST = 34.970	ANG DISPL = 66.788 DEG
OFFTRACKING = 2.687	DIST = 36.970	ANG DISPL = 70.607 DEG
OFFTRACKING = 2.701	DIST = 38.970	ANG DISPL = 74.427 DEG
OFFTRACKING = 2.714	DIST = 40.970	ANG DISPL = 78.247 DEG
OFFTRACKING = 2.724	DIST = 42.970	ANG DISPL = 82.067 DEG
OFFTRACKING = 2.733	DIST = 44.970	ANG DISPL = 85.886 DEG
OFFTRACKING = 2.741	DIST = 46.970	ANG DISPL = 89.706 DEG
OFFTRACKING = 2.747	DIST = 48.970	ANG DISPL = 93.526 DEG
OFFTRACKING = 2.753	DIST = 50.970	ANG DISPL = 97.346 DEG
OFFTRACKING = 2.758	DIST = 52.970	ANG DISPL = 101.165 DEG
OFFTRACKING = 2.762	DIST = 54.970	ANG DISPL = 104.985 DEG
OFFTRACKING = 2.765	DIST = 56.970	ANG DISPL = 108.805 DEG
OFFTRACKING = 2.768	DIST = 58.970	ANG DISPL = 112.624 DEG
OFFTRACKING = 2.770	DIST = 60.970	ANG DISPL = 116.444 DEG

Problem (3) A-TRAIN WITH 3 TRAILERS 8/12/83

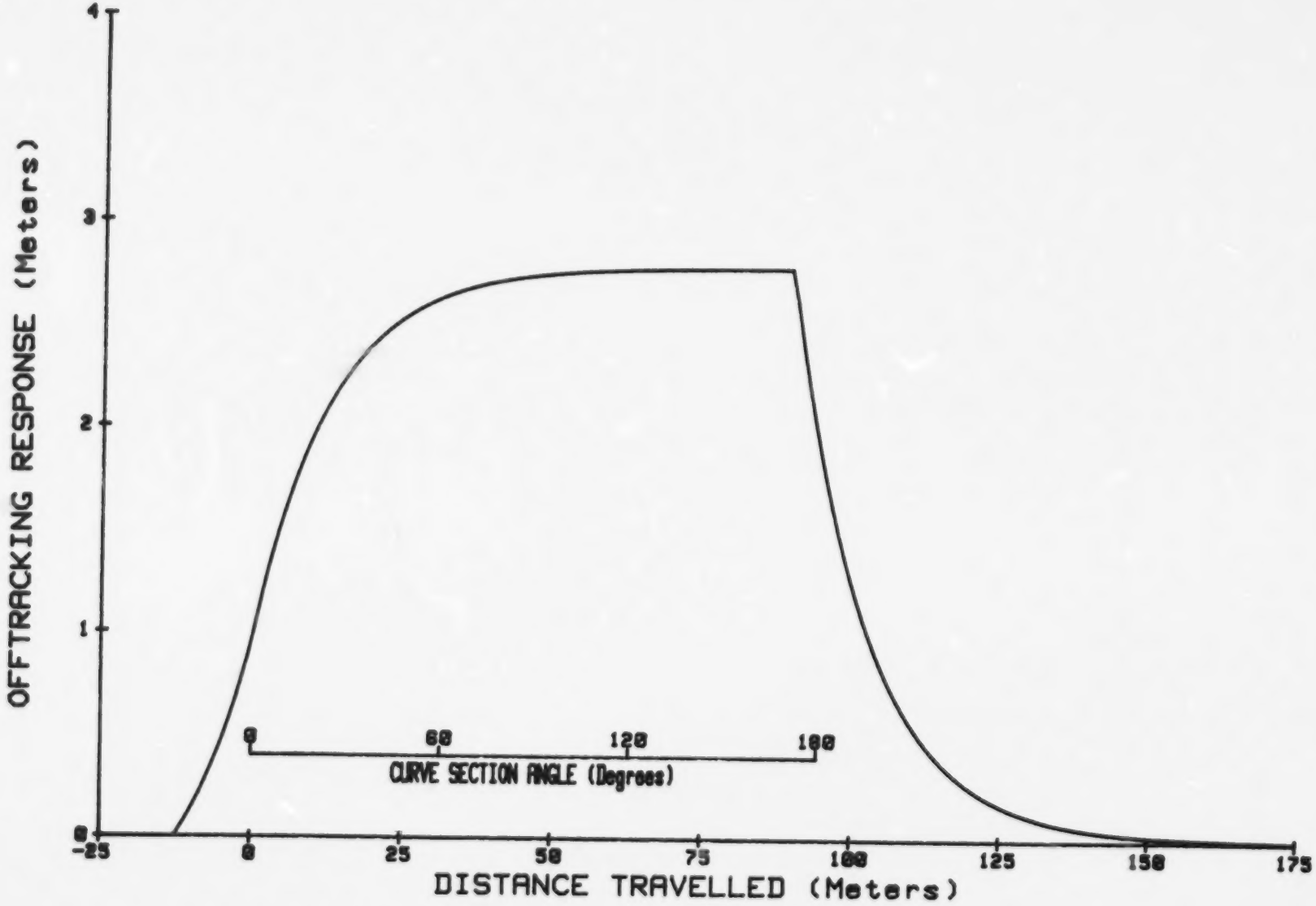
OFFTRACKING = 2.773	DIST = 62.970	ANG DISPL = 120.264 DEG
OFFTRACKING = 2.774	DIST = 64.970	ANG DISPL = 124.084 DEG
OFFTRACKING = 2.776	DIST = 66.970	ANG DISPL = 127.903 DEG
OFFTRACKING = 2.777	DIST = 68.970	ANG DISPL = 131.723 DEG
OFFTRACKING = 2.778	DIST = 70.970	ANG DISPL = 135.543 DEG
OFFTRACKING = 2.779	DIST = 72.970	ANG DISPL = 139.362 DEG
OFFTRACKING = 2.780	DIST = 74.970	ANG DISPL = 143.182 DEG
OFFTRACKING = 2.781	DIST = 76.970	ANG DISPL = 147.002 DEG
OFFTRACKING = 2.782	DIST = 78.970	ANG DISPL = 150.822 DEG
OFFTRACKING = 2.782	DIST = 80.970	ANG DISPL = 154.641 DEG
OFFTRACKING = 2.783	DIST = 82.970	ANG DISPL = 158.461 DEG
OFFTRACKING = 2.783	DIST = 84.970	ANG DISPL = 162.281 DEG
OFFTRACKING = 2.783	DIST = 86.970	ANG DISPL = 166.100 DEG
OFFTRACKING = 2.784	DIST = 88.970	ANG DISPL = 169.920 DEG
OFFTRACKING = 2.532	DIST = 90.970	ANG DISPL = 173.740 DEG
OFFTRACKING = 2.161	DIST = 92.970	ANG DISPL = 177.560 DEG
OFFTRACKING = 1.844	DIST = 94.970	RUN OUT TANGENT
OFFTRACKING = 1.574	DIST = 96.970	RUN OUT TANGENT
OFFTRACKING = 1.344	DIST = 98.970	RUN OUT TANGENT
OFFTRACKING = 1.147	DIST = 100.970	RUN OUT TANGENT
OFFTRACKING = .979	DIST = 102.970	RUN OUT TANGENT
OFFTRACKING = .835	DIST = 104.970	RUN OUT TANGENT
OFFTRACKING = .713	DIST = 106.970	RUN OUT TANGENT
OFFTRACKING = .608	DIST = 108.970	RUN OUT TANGENT
OFFTRACKING = .519	DIST = 110.970	RUN OUT TANGENT
OFFTRACKING = .443	DIST = 112.970	RUN OUT TANGENT
OFFTRACKING = .378	DIST = 114.970	RUN OUT TANGENT
OFFTRACKING = .323	DIST = 116.970	RUN OUT TANGENT
OFFTRACKING = .276	DIST = 118.970	RUN OUT TANGENT
OFFTRACKING = .235	DIST = 120.970	RUN OUT TANGENT
OFFTRACKING = .201	DIST = 122.970	RUN OUT TANGENT
OFFTRACKING = .171	DIST = 124.970	RUN OUT TANGENT
OFFTRACKING = .146	DIST = 126.970	RUN OUT TANGENT
OFFTRACKING = .125	DIST = 128.970	RUN OUT TANGENT
OFFTRACKING = .106	DIST = 130.970	RUN OUT TANGENT
OFFTRACKING = .091	DIST = 132.970	RUN OUT TANGENT
OFFTRACKING = .078	DIST = 134.970	RUN OUT TANGENT
OFFTRACKING = .066	DIST = 136.970	RUN OUT TANGENT
OFFTRACKING = .057	DIST = 138.970	RUN OUT TANGENT
OFFTRACKING = .048	DIST = 140.970	RUN OUT TANGEN
OFFTRACKING = .041	DIST = 142.970	RUN OUT TANGENT
OFFTRACKING = .035	DIST = 144.970	RUN OUT TANGENT
OFFTRACKING = .030	DIST = 146.970	RUN OUT TANGENT
OFFTRACKING = .026	DIST = 148.970	RUN OUT TANGENT
OFFTRACKING = .022	DIST = 150.970	RUN OUT TANGENT

THE OFFTRACKING VALUES LISTED ARE DEFINED AS THE LATERAL SHIFT OF THE LAST AXLE GROUP OF THE REAR TRAILER WITH RESPECT TO THE PATH FOLLOWED BY THE STEERING AXLE OF THE TRACTOR. ALL POINTS ARE MEASURED FROM THE CENTER LINE OF THE VEHICLE AXIS.

DIST IS THE POINT ON THE PATH WHERE THE GIVEN OFFTRACK VALUE APPLIES. DIST=0 REFERS TO THE START OF THE CURVE. THEREFORE THE LEAD TANGENT PORTION IS NEGATIVE.

ANG DISPL IS THE ANGULAR DISPLACEMENT OF THE CONSTANT RADIUS CURVE WHERE ZERO IS AT THE BEGINNING OF THE CURVE.

********** All units are in METERS **********



GRAPHIC OUTPUT EXAMPLE 3 – TRIPLE TRAIN (30 m RADIUS – 180 DEG. TURN)

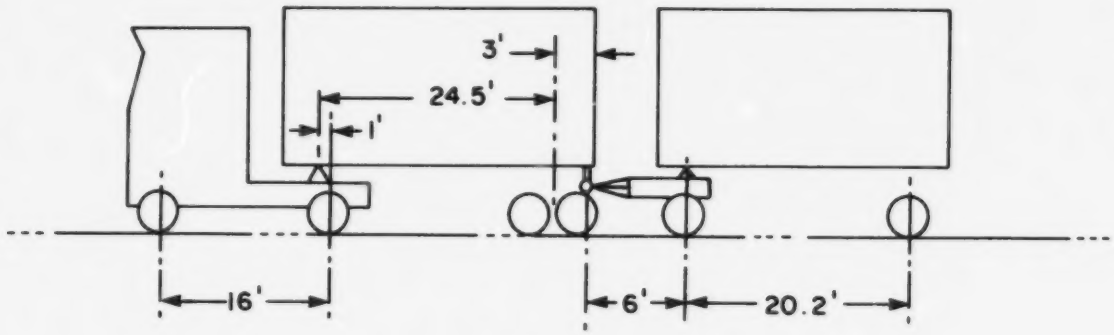
Example 4

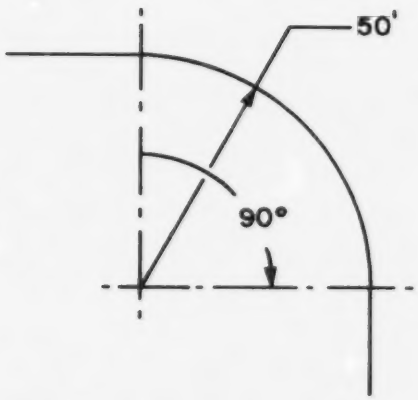
Problem: Statement

Describe the transient off tracking of the illustrated C-train for:

a) free castering B-dolly

b) B-dolly with a locked steering axle through a 50 ft constant radius curve of 90° duration with a tangent lead in and run out.





VEHICLE DIMENSIONS AND ROAD ALIGNMENT FOR EXAMPLE 4

Solution

Alignment Specifications

- curve radius (R) 50 ft at centre of tractor's steering axle
- curve duration 90°, $\theta = \pi/2$ radians
- tangent lead in and run out

Vehicle Specifications

- tractor wheelbase (wb_1) = 16 ft
- trailer wheelbase (wb_2) = 24.5 ft
- tractor fifth wheel setting (d_1) = +1 ft
- 1st trailer axle spread (D1) = 3 ft
- 1st trailer rear overhang (d_2) = 3 ft
- 1st B-dolly hitch length (wb_3) = 6 ft
- 2nd trailer wheelbase (wb_4) = 20.2 ft

Solution 4(A) — free castering B-dolly

EQUATION

$$wb_T = (16 - 1 + 24.5 + 3 + 6 + 20.2) \quad \text{(A)}$$

$$wb_T = 68.7 \text{ ft}$$

Effective rear overhang $= d_2 + wb_3$

$= 3 + 6$

$= 9$ ft

$$wb_e = \sqrt{16^2 + 24.5^2 + 20.2^2 - \left((-1)^2 + 9^2\right)} \quad \text{(B)}$$

$$wb_e = 34.4 \text{ ft}$$

$$Z = 0.6\left(\frac{68.7 \times 34.4}{68.7 + 34.4}\right) \quad \text{(C)}$$

$$Z = 13.8 \text{ ft}$$

$$OT_{ss} = 50 - \sqrt{50^2 - (34.4)^2} \quad \text{(D)}$$

$$OT_{ss} = 13.7 \text{ ft}$$

$$C = 34.4\,\ell n\left[(34.4^2 + 2 \times 50 \times 13.7 - 13.7^2)\frac{\exp(-1)}{2 \times 50 \times 13.7}\right] + \frac{50\pi}{2} \quad \text{(E)}$$

$$C = 62.9$$

$$OT_{max} = 13.7 - 13.7 \exp [-(62.9 + 13.8)/34.4] \quad \text{(F)}$$

$$OT_{max} = 12.2 \text{ ft}$$

$$OT_{so} = 13.7 - 13.7 \exp (-13.8/34.4) \quad \text{(G)}$$

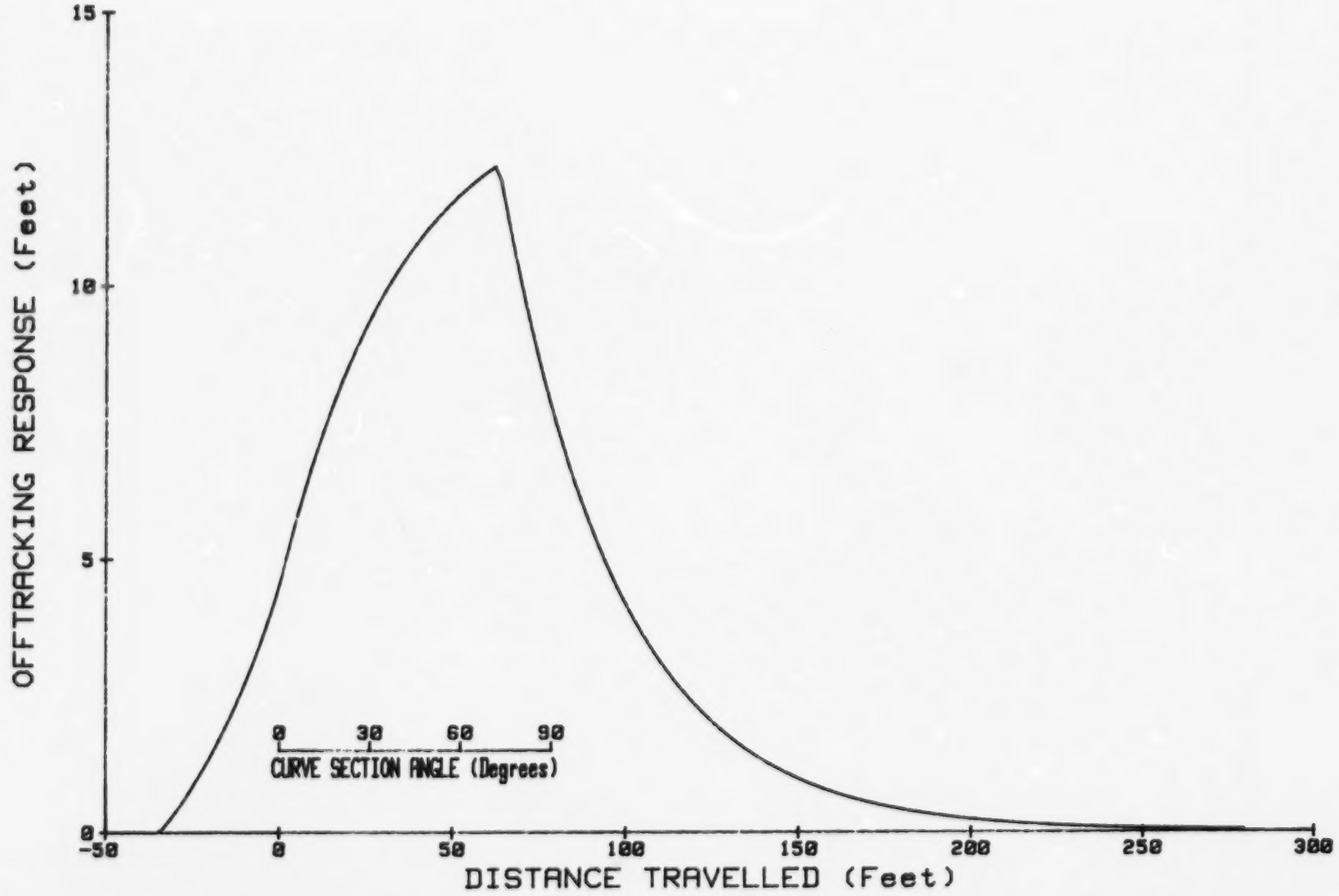
$$OT_{so} = 4.53 \text{ ft}$$

Having solved for the constants, the three piecewise general off tracking equations become:

$$OT_1 = -0.582\,(4.53) + 1.582\,(4.53) \exp (S/34.4) \Big|_{S=-34.4}^{0}$$

$$OT_2 = 13.7 - 13.7 \exp [-(S+13.8)/34.4] \Big|_{S=0}^{62.9}$$

$$OT_3 = 12.2 \exp [(-S+62.9)/34.4] \Big|_{S=62.9}^{-}$$



GRAPHIC OUTPUT EXAMPLE 4(A) – C-TRAIN – FREE STEERING B-DOLLY (50 FT RADIUS 90 DEG. TURN)

Problem 4(a) C-TRAIN with FREE CASTERING B-DOLLY 14/12/83

****

ROAD AND VEHICLE SPECIFICATIONS

CURVE DURATION = 90.00 Degrees (1.57 Radians)
CURVE RADIUS = 50.00 Ft

TRACTOR WHEELBASE= 16.00 Ft
FIRST TRAILER WHEELBASE= 24.50 Ft
FIRST DOLLY HITCH LENGTH= 0.00 Ft
SECOND TRAILER OVERHANG= 0.00 Ft
THIRD TRAILER WHEELBASE= 0.00 Ft
THIRD DOLLY HITCH LENGTH= 0.00 Ft

TRACTOR FIFTH WHEEL POSITION= 1.00 Ft
FIRST TRAILER OVERHANG= 9.00 Ft
SECOND TRAILER WHEELBASE= 20.20 Ft
SECOND DOLLY HITCH LENGTH= 0.00 Ft
THIRD TRAILER OVERHANG= 0.00 Ft
FOURTH TRAILER WHEELBASE 0.00 Ft

****

STEADY STATE OFF TRACKING = 13.70 Ft TOTAL WHEEL BASE = 68.70 Ft
EFFECTIVE WHEEL BASE = 34.38 Ft

MAXIMUM OFF-TRACKING = 12.23 Ft occurs 62.92 Ft at 72.10 Degrees
from the start of the curve.

^^^^^^^^^^

OFFTRACKING = 000	DIST =-68.700	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-66.700	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-64.700	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-62.700	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-60.700	LEAD IN TANGEN
OFFTRACKING = 000	DIST =-58.700	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-56.700	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-54.700	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-52.700	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-50.700	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-48.700	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-46.700	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-44.700	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-42.700	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-40.700	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-38.700	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-36.700	LEAD IN TANGENT
OFFTRACKING = 000	DIST =-34.700	LEAD IN TANGENT
OFFTRACKING = .132	DIST =-32.700	LEAD IN TANGENT EXP FUNC
OFFTRACKING = .297	DIST =-30.700	LEAD IN TANGENT EXP FUNC
OFFTRACKING = .472	DIST =-28.700	LEAD IN TANGENT EXP FUNC
OFFTRACKING = .658	DIST =-26.700	LEAD IN TANGENT EXP FUNC
OFFTRACKING = .855	DIST =-24.700	LEAD IN TANGENT EXP FUNC
OFFTRACKING = 1.063	DIST =-22.700	LEAD IN TANGENT EXP FUNC
OFFTRACKING = 1.284	DIST =-20.700	LEAD IN TANGENT EXP FUNC
OFFTRACKING = 1.519	DIST =-18.700	LEAD IN TANGENT EXP FUNC
OFFTRACKING = 1.767	DIST =-16.700	LEAD IN TANGENT EXP FUNC
OFFTRACKING = 2.030	DIST =-14.700	LEAD IN TANGENT EXP FUNC
OFFTRACKING = 2.309	DIST =-12.700	LEAD IN TANGENT EXP FUNC
OFFTRACKING = 2.605	DIST =-10.700	LEAD IN TANGENT EXP FUNC
OFFTRACKING = 2.918	DIST =-8.700	LEAD IN TANGENT EXP FUNC
OFFTRACKING = 3.251	DIST =-6.700	LEAD IN TANGENT EXP FUNC
OFFTRACKING = 3.603	DIST =-4.700	LEAD IN TANGENT EXP FUNC
OFFTRACKING = 3.976	DIST =-2.700	LEAD IN TANGENT EXP FUNC
OFFTRACKING = 4.371	DIST =-.700	LEAD IN TANGENT EXP FUNC
OFFTRACKING = 4.856	DIST = 1.300	ANG DISPL = 1.490 DEG
OFFTRACKING = 5.356	DIST = 3.300	ANG DISPL = 3.782 DEG
OFFTRACKING = 5.827	DIST = 5.300	ANG DISPL = 6.073 DEG
OFFTRACKING = 6.272	DIST = 7.300	ANG DISPL = 8.365 DEG
OFFTRACKING = 6.692	DIST = 9.300	ANG DISPL = 10.657 DEG
OFFTRACKING = 7.088	DIST = 11.300	ANG DISPL = 12.949 DEG
OFFTRACKING = 7.461	DIST = 13.300	ANG DISPL = 15.241 DEG
OFFTRACKING = 7.814	DIST = 15.300	ANG DISPL = 17.533 DEG
OFFTRACKING = 8.147	DIST = 17.300	ANG DISPL = 19.824 DEG
OFFTRACKING = 8.460	DIST = 19.300	ANG DISPL = 22.116 DEG
OFFTRACKING = 8.756	DIST = 21.300	ANG DISPL = 24.408 DEG

Problem 4(a) C-TRAIN with FREE CASTERING B-DOLLY 14/12/83

OFFTRACKING = 9.036	DIST = 23.300	ANG DISPL = 26.700 DEG
OFFTRACKING = 9.299	DIST = 25.300	ANG DISPL = 28.992 DEG
OFFTRACKING = 9.548	DIST = 27.300	ANG DISPL = 31.283 DEG
OFFTRACKING = 9.783	DIST = 29.300	ANG DISPL = 33.575 DEG
OFFTRACKING = 10.004	DIST = 31.300	ANG DISPL = 35.867 DEG
OFFTRACKING = 10.213	DIST = 33.300	ANG DISPL = 38.159 DEG
OFFTRACKING = 10.410	DIST = 35.300	ANG DISPL = 40.451 DEG
OFFTRACKING = 10.596	DIST = 37.300	ANG DISPL = 42.743 DEG
OFFTRACKING = 10.771	DIST = 39.300	ANG DISPL = 45.034 DEG
OFFTRACKING = 10.937	DIST = 41.300	ANG DISPL = 47.326 DEG
OFFTRACKING = 11.093	DIST = 43.300	ANG DISPL = 49.618 DEG
OFFTRACKING = 11.240	DIST = 45.300	ANG DISPL = 51.910 DEG
OFFTRACKING = 11.379	DIST = 47.300	ANG DISPL = 54.202 DEG
OFFTRACKING = 11.510	DIST = 49.300	ANG DISPL = 56.494 DEG
OFFTRACKING = 11.634	DIST = 51.300	ANG DISPL = 58.785 DEG
OFFTRACKING = 11.751	DIST = 53.300	ANG DISPL = 61.077 DEG
OFFTRACKING = 11.861	DIST = 55.300	ANG DISPL = 63.369 DEG
OFFTRACKING = 11.965	DIST = 57.300	ANG DISPL = 65.661 DEG
OFFTRACKING = 12.063	DIST = 59.300	ANG DISPL = 67.953 DEG
OFFTRACKING = 12.155	DIST = 61.300	ANG DISPL = 70.245 DEG
OFFTRACKING = 12.093	DIST = 63.300	ANG DISPL = 72.536 DEG
OFFTRACKING = 11.410	DIST = 65.300	ANG DISPL = 74.828 DEG
OFFTRACKING = 10.765	DIST = 67.300	ANG DISPL = 77.120 DEG
OFFTRACKING = 10.157	DIST = 69.300	ANG DISPL = 79.412 DEG
OFFTRACKING = 9.583	DIST = 71.300	ANG DISPL = 81.704 DEG
OFFTRACKING = 9.041	DIST = 73.300	ANG DISPL = 83.996 DEG
OFFTRACKING = 8.530	DIST = 75.300	ANG DISPL = 86.287 DEG
OFFTRACKING = 8.048	DIST = 77.300	ANG DISPL = 88.579 DE
OFFTRACKING = 7.594	DIST = 79.300	RUN OUT TANGENT
OFFTRACKING = 7.164	DIST = 81.300	RUN OUT TANGENT
OFFTRACKING = 6.760	DIST = 83.300	RUN OUT TANGENT
OFFTRACKING = 6.378	DIST = 85.300	RUN OUT TANGENT
OFFTRACKING = 6.017	DIST = 87.300	RUN OUT TANGENT
OFFTRACKING = 5.677	DIST = 89.300	RUN OUT TANGENT
OFFTRACKING = 5.356	DIST = 91.300	RUN OUT TANGENT
OFFTRACKING = 5.054	DIST = 93.300	RUN OUT TANGENT
OFFTRACKING = 4.768	DIST = 95.300	RUN OUT TANGENT
OFFTRACKING = 4.499	DIST = 97.300	RUN OUT TANGENT
OFFTRACKING = 4.245	DIST = 99.300	RUN OUT TANGENT
OFFTRACKING = 4.005	DIST = 101.300	RUN OUT TANGENT
OFFTRACKING = 3.778	DIST = 103.300	RUN OUT TANGENT
OFFTRACKING = 3.565	DIST = 105.300	RUN OUT TANGENT
OFFTRACKING = 3.363	DIST = 107.300	RUN OUT TANGENT
OFFTRACKING = 3.173	DIST = 109.300	RUN OUT TANGENT
OFFTRACKING = 2.994	DIST = 111.300	RUN OUT TANGENT
OFFTRACKING = 2.825	DIST = 113.300	RUN OUT TANGENT
OFFTRACKING = 2.665	DIST = 115.300	RUN OUT TANGENT
OFFTRACKING = 2.515	DIST = 117.300	RUN OUT TANGENT
OFFTRACKING = 2.373	DIST = 119.300	RUN OUT TANGENT
OFFTRACKING = 2.239	DIST = 121.300	RUN OUT TANGENT
OFFTRACKING = 2.112	DIST = 123.300	RUN OUT TANGENT
OFFTRACKING = 1.993	DIST = 125.300	RUN OUT TANGENT
OFFTRACKING = 1.880	DIST = 127.300	RUN OUT TANGENT
OFFTRACKING = 1.774	DIST = 129.300	RUN OUT TANGENT
OFFTRACKING = 1.674	DIST = 131.300	RUN OUT TANGENT
OFFTRACKING = 1.579	DIST = 133.300	RUN OUT TANGENT
OFFTRACKING = 1.490	DIST = 135.300	RUN OUT TANGENT
OFFTRACKING = 1.406	DIST = 137.300	RUN OUT TANGENT
OFFTRACKING = 1.326	DIST = 139.300	RUN OUT TANGENT
OFFTRACKING = 1.251	DIST = 141.300	RUN OUT TANGENT
OFFTRACKING = 1.181	DIST = 143.300	RUN OUT TANGENT
OFFTRACKING = 1.114	DIST = 145.300	RUN OUT TANGENT
OFFTRACKING = 1.051	DIST = 147.300	RUN OUT TANGENT
OFFTRACKING = .992	DIST = 149.300	RUN OUT TANGENT
OFFTRACKING = .936	DIST = 151.300	RUN OUT TANGENT
OFFTRACKING = .883	DIST = 153.300	RUN OUT TANGENT
OFFTRACKING = .833	DIST = 155.300	RUN OUT TANGENT
OFFTRACKING = .786	DIST = 157.300	RUN OUT TANGENT
OFFTRACKING = .741	DIST = 159.300	RUN OUT TANGENT
OFFTRACKING = .699	DIST = 161.300	RUN OUT TANGENT
OFFTRACKING = .660	DIST = 163.300	RUN OUT TANGENT
OFFTRACKING = .623	DIST = 165.300	RUN OUT TANGENT

Problem 4(a) C-TRAIN with FREE CASTERING B-DOLLY 14/12/83

OFFTRACKING = .587	DIST = 167.300	RUN OUT TANGENT
OFFTRACKING = .554	DIST = 169.300	RUN OUT TANGENT
OFFTRACKING = .523	DIST = 171.300	RUN OUT TANGENT
OFFTRACKING = .493	DIST = 173.300	RUN OUT TANGENT
OFFTRACKING = .465	DIST = 175.300	RUN OUT TANGENT
OFFTRACKING = .439	DIST = 177.300	RUN OUT TANGENT
OFFTRACKING = .414	DIST = 179.300	RUN OUT TANGENT
OFFTRACKING = .391	DIST = 181.300	RUN OUT TANGENT
OFFTRACKING = .369	DIST = 183.300	RUN OUT TANGENT
OFFTRACKING = .348	DIST = 185.300	RUN OUT TANGENT
OFFTRACKING = .328	DIST = 187.300	RUN OUT TANGENT
OFFTRACKING = .310	DIST = 189.300	RUN OUT TANGENT
OFFTRACKING = .292	DIST = 191.300	RUN OUT TANGENT
OFFTRACKING = .276	DIST = 193.300	RUN OUT TANGENT
OFFTRACKING = .260	DIST = 195.300	RUN OUT TANGENT
OFFTRACKING = .245	DIST = 197.300	RUN OUT TANGENT
OFFTRACKING = .232	DIST = 199.300	RUN OUT TANGENT
OFFTRACKING = .219	DIST = 201.300	RUN OUT TANGENT
OFFTRACKING = .206	DIST = 203.300	RUN OUT TANGENT
OFFTRACKING = .195	DIST = 205.300	RUN OUT TANGENT
OFFTRACKING = .184	DIST = 207.300	RUN OUT TANGENT
OFFTRACKING = .173	DIST = 209.300	RUN OUT TANGENT
OFFTRACKING = .163	DIST = 211.300	RUN OUT TANGENT
OFFTRACKING = .154	DIST = 213.300	RUN OUT TANGENT
OFFTRACKING = .145	DIST = 215.300	RUN OUT TANGENT

THE OFFTRACKING VALUES LISTED ARE DEFINED AS THE LATERAL SHIFT OF THE LAST AXLE GROUP OF THE REAR TRAILER WITH RESPECT TO THE PATH FOLLOWED BY THE STEERING AXLE OF THE TRACTOR. ALL POINTS ARE MEASURED FROM THE CENTER LINE OF THE VEHICLE AXIS.

DIST IS THE POINT ON THE PATH WHERE THE GIVEN OFFTRACK VALUE APPLIES. DIST=0 REFERS TO THE START OF THE CURVE. THEREFORE THE LEAD TANGENT PORTION IS NEGATIVE.

ANG DISPL IS THE ANGULAR DISPLACEMENT OF THE CONSTANT RADIUS CURVE WHERE ZERO IS AT THE BEGINNING OF THE CURVE.

********** All units are in FEET **********

Solution 4(B) — B-dolly with locked steering axle.

Imaginary wheelbase:

EQUATION

$$wb_{IMAG} = 24.5 + (3 + 2(75))/6 \qquad \text{(D2)}$$

$$wb_{IMAG} = 27.5 \text{ ft}$$

$$wb_T = (16 - 1 + 27.5 + 6 + 20.2) \qquad \text{(A)}$$

$$wb_T = 68.7 \text{ ft}$$

Effective rear overhang $D_i = wb_3 = 6$ ft

$$wb_e = \sqrt{16^2 + 27.5^2 + 20.2^2 - ((-1)^2 + 6^2)} \qquad \text{(B)}$$

$$wb_e = 37.2 \text{ ft}$$

$$Z = 0.6 \left(\frac{68.7 \times 37.2}{68.7 + 37.2} \right) \qquad \text{(C)}$$

$$Z = 14.5 \text{ ft}$$

$$OT_{ss} = 50 - \sqrt{50^2 - 37.2^2} \qquad \text{(D)}$$

$$OT_{ss} = 16.6 \text{ ft}$$

$$C = 37.2 \, \ell n \left[(37.2^2 + 2 \times 50 \times 16.6 - 16.6^2) \, \frac{\exp(-1)}{2 \times 50 \times 16.6} \right] + 50 \frac{\pi}{2} \qquad \text{(E)}$$

$$C = 60.4$$

$$OT_{max} = 16.6 - 16.6 \exp \, [-(60.4 + 14.5)/37.2] \qquad \text{(F)}$$

$$OT_{max} = 14.4 \text{ ft}$$

$$OT_{so} = 16.6 - 16.6 \exp \, (-14.5/37.2) \qquad \text{(G)}$$

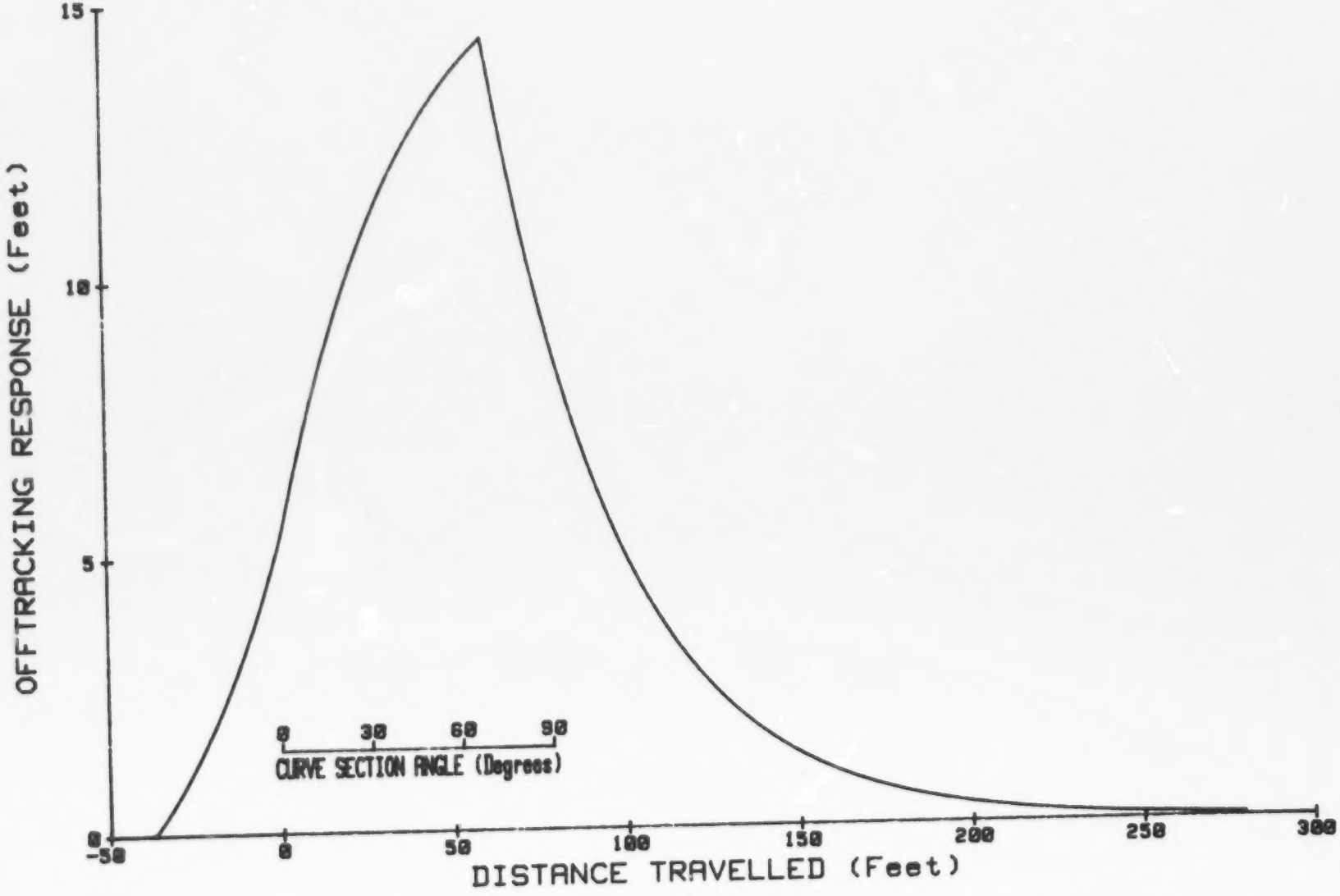
$$OT_{so} = 5.36 \text{ ft}$$

Having solved for the constants, the three piecewise continuous tracking equations become:

$$OT_1 = -0.582\,(5.36) + 1.582\,(5.36)\exp(S/37.2) \Big|_{S=-37.2}^{0}$$

$$OT_2 = 16.6 - 16.6\exp[(-S+14.5)/37.2] \Big|_{S=0}^{60.4}$$

$$OT_3 = 14.4\exp[(-S+60.4)/37.2] \Big|_{S=60.4}^{-}$$



GRAPHIC OUTPUT EXAMPLE 4(B) – C-TRAIN – LOCKED STEERING B-DOLLY (50 FT RADIUS 90 DEG. TURN)

```
Problem 4(h)    C-TRAIN with A LOCKED STEERING B-DOLLY    14/12/83

**********************************************************************

          ROAD AND VEHICLE SPECIFICATIONS

CURVE DURATION = 90.00  Degrees ( 1.57  Radians)
CURVE RADIUS = 50.00 Ft

TRACTOR WHEELBASE= 16.00 Ft          TRACTOR FIFTH WHEEL POSITION= 1.00 Ft
FIRST TRAILER WHEELBASE= 27.50 Ft    FIRST TRAILER OVERHANG= 6.00 Ft
FIRST DOLLY HITCH LENGTH= 0.00 Ft    SECOND TRAILER WHEELBASE= 20.20 Ft
SECOND TRAILER OVERHANG= 0.00 Ft     SECOND DOLLY HITCH LENGTH= 0.00 Ft
THIRD TRAILER WHEELBASE= 0.00 Ft     THIRD TRAILER OVERHANG= 0.00 Ft
THIRD DOLLY HITCH LENGTH= 0.00 Ft    FOURTH TRAILER WHEELBASE 0.00 Ft

**********************************************************************

STEADY STATE OFF TRACKING = 16.58 Ft  TOTAL WHEEL BASE = 68.70 Ft
EFFECTIVE WHEEL BASE = 37.19 Ft

MAXIMUM OFF-TRACKING = 14.37 Ft occurs  60.38 Ft at 69.19 Degrees
from the start of the curve.

^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
 OFFTRACKING =  000        DIST =-68.700     LEAD IN TANGENT
 OFFTRACKING =  000        DIST =-66.700     LEAD IN TANGENT
 OFFTRACKING =  000        DIST =-64.700     LEAD IN TANGENT
 OFFTRACKING =  000        DIST =-62.700     LEAD IN TANGENT
 OFFTRACKING =  000        DIST =-60.700     LEAD IN TANGENT
 OFFTRACKING =  000        DIST =-58.700     LEAD IN TANGENT
 OFFTRACKING =  000        DIST =-56.700     LEAD IN TANGENT
 OFFTRACKING =  000        DIST =-54.700     LEAD IN TANGENT
 OFFTRACKING =  000        DIST =-52.700     LEAD IN TANGENT
 OFFTRACKING =  000        DIST =-50.700     LEAD IN TANGENT
 OFFTRACKING =  000        DIST =-48.700     LEAD IN TANGENT
 OFFTRACKING =  000        DIST =-46.700     LEAD IN TANGENT
 OFFTRACKING =  000        DIST =-44.700     LEAD IN TANGENT
 OFFTRACKING =  000        DIST =-42.700     LEAD IN TANGENT
 OFFTRACKING =  000        DIST =-40.700     LEAD IN TANGENT
 OFFTRACKING =  000        DIST =-38.700     LEAD IN TANGENT
 OFFTRACKING =  .041       DIST =-36.700     LEAD IN TANGENT EXP FUNC
 OFFTRACKING =  .216       DIST =-34.700     LEAD IN TANGENT EXP FUNC
 OFFTRACKING =  .399       DIST =-32.700     LEAD IN TANGENT EXP FUNC
 OFFTRACKING =  .593       DIST =-30.700     LEAD IN TANGENT EXP FUNC
 OFFTRACKING =  .798       DIST =-28.700     LEAD IN TANGENT EXP FUNC
 OFFTRACKING =  1.014      DIST =-26.700     LEAD IN TANGENT EXP FUNC
 OFFTRACKING =  1.242      DIST =-24.700     LEAD IN TANGENT EXP FUNC
 OFFTRACKING =  1.483      DIST =-22.700     LEAD IN TANGENT EXP FUNC
 OFFTRACKING =  1.737      DIST =-20.700     LEAD IN TANGENT EXP FUNC
 OFFTRACKING =  2.004      DIST =-18.700     LEAD IN TANGENT EXP FUNC
 OFFTRACKING =  2.287      DIST =-16.700     LEAD IN TANGENT EXP FUNC
 OFFTRACKING =  2.585      DIST =-14.700     LEAD IN TANGENT EXP FUNC
 OFFTRACKING =  2.900      DIST =-12.700     LEAD IN TANGENT EXP FUNC
 OFFTRACKING =  3.232      DIST =-10.700     LEAD IN TANGENT EXP FUNC
 OFFTRACKING =  3.583      DIST =-8.700      LEAD IN TANGENT EXP FUNC
 OFFTRACKING =  3.953      DIST =-6.700      LEAD IN TANGENT EXP FUNC
 OFFTRACKING =  4.343      DIST =-4.700      LEAD IN TANGENT EXP FUNC
 OFFTRACKING =  4.755      DIST =-2.700      LEAD IN TANGENT EXP FUNC
 OFFTRACKING =  5.189      DIST =-.700       LEAD IN TANGENT EXP FUNC
 OFFTRACKING =  5.733      DIST =  1.300     ANG DISPL = 1.490 DEG
 OFFTRACKING =  6.301      DIST =  3.300     ANG DISPL = 3.782 DEG
 OFFTRACKING =  6.839      DIST =  5.300     ANG DISPL = 6.073 DEG
 OFFTRACKING =  7.349      DIST =  7.300     ANG DISPL = 8.365 DEG
 OFFTRACKING =  7.833      DIST =  9.300     ANG DISPL = 10.657 DEG
 OFFTRACKING =  8.291      DIST =  11.300    ANG DISPL = 12.949 DEG
 OFFTRACKING =  8.725      DIST =  13.300    ANG DISPL = 15.241 DEG
 OFFTRACKING =  9.136      DIST =  15.300    ANG DISPL = 17.533 DEG
 OFFTRACKING =  9.526      DIST =  17.300    ANG DISPL = 19.824 DEG
 OFFTRACKING =  9.896      DIST =  19.300    ANG DISPL = 22.116 DEG
 OFFTRACKING =  10.246     DIST =  21.300    ANG DISPL = 24.408 DEG
```

Problem 4(b) C-TRAIN with A LOCKED STEERING B-DOLLY 14/12/83

OFFTRACKING = 10.578	DIST = 23.300	ANG DISPL = 26.700 DEG
OFFTRACKING = 10.892	DIST = 25.300	ANG DISPL = 28.992 DEG
OFFTRACKING = 11.190	DIST = 27.300	ANG DISPL = 31.283 DEG
OFFTRACKING = 11.472	DIST = 29.300	ANG DISPL = 33.575 DEG
OFFTRACKING = 11.740	DIST = 31.300	ANG DISPL = 35.867 DEG
OFFTRACKING = 11.993	DIST = 33.300	ANG DISPL = 38.159 DEG
OFFTRACKING = 12.234	DIST = 35.300	ANG DISPL = 40.451 DEG
OFFTRACKING = 12.461	DIST = 37.300	ANG DISPL = 42.743 DEG
OFFTRACKING = 12.677	DIST = 39.300	ANG DISPL = 45.034 DEG
OFFTRACKING = 12.882	DIST = 41.300	ANG DISPL = 47.326 DEG
OFFTRACKING = 13.075	DIST = 43.300	ANG DISPL = 49.618 DEG
OFFTRACKING = 13.259	DIST = 45.300	ANG DISPL = 51.910 DEG
OFFTRACKING = 13.433	DIST = 47.300	ANG DISPL = 54.202 DEG
OFFTRACKING = 13.598	DIST = 49.300	ANG DISPL = 56.494 DEG
OFFTRACKING = 13.754	DIST = 51.300	ANG DISPL = 58.785 DEG
OFFTRACKING = 13.902	DIST = 53.300	ANG DISPL = 61.077 DEG
OFFTRACKING = 14.043	DIST = 55.300	ANG DISPL = 63.369 DEG
OFFTRACKING = 14.176	DIST = 57.300	ANG DISPL = 65.661 DEG
OFFTRACKING = 14.302	DIST = 59.300	ANG DISPL = 67.953 DEG
OFFTRACKING = 14.017	DIST = 61.300	ANG DISPL = 70.245 DEG
OFFTRACKING = 13.284	DIST = 63.300	ANG DISPL = 72.536 DEG
OFFTRACKING = 12.588	DIST = 65.300	ANG DISPL = 74.828 DEG
OFFTRACKING = 11.929	DIST = 67.300	ANG DISPL = 77.120 DEG
OFFTRACKING = 11.305	DIST = 69.300	ANG DISPL = 79.412 DEG
OFFTRACKING = 10.713	DIST = 71.300	ANG DISPL = 81.704 DEG
OFFTRACKING = 10.152	DIST = 73.300	ANG DISPL = 83.996 DEG
OFFTRACKING = 9.620	DIST = 75.300	ANG DISPL = 86.287 DEG
OFFTRACKING = 9.117	DIST = 77.300	ANG DISPL = 88.579 DEG
OFFTRACKING = 8.639	DIST = 79.300	RUN OUT TANGENT
OFFTRACKING = 8.187	DIST = 81.300	RUN OUT TANGENT
OFFTRACKING = 7.758	DIST = 83.300	RUN OUT TANGENT
OFFTRACKING = 7.352	DIST = 85.300	RUN OUT TANGENT
OFFTRACKING = 6.967	DIST = 87.300	RUN OUT TANGENT
OFFTRACKING = 6.603	DIST = 89.300	RUN OUT TANGENT
OFFTRACKING = 6.257	DIST = 91.300	RUN OUT TANGENT
OFFTRACKING = 5.929	DIST = 93.300	RUN OUT TANGENT
OFFTRACKING = 5.619	DIST = 95.300	RUN OUT TANGENT
OFFTRACKING = 5.325	DIST = 97.300	RUN OUT TANGENT
OFFTRACKING = 5.046	DIST = 99.300	RUN OUT TANGENT
OFFTRACKING = 4.782	DIST = 101.300	RUN OUT TANGENT
OFFTRACKING = 4.531	DIST = 103.300	RUN OUT TANGENT
OFFTRACKING = 4.294	DIST = 105.300	RUN OUT TANGENT
OFFTRACKING = 4.069	DIST = 107.300	RUN OUT TANGENT
OFFTRACKING = 3.856	DIST = 109.300	RUN OUT TANGENT
OFFTRACKING = 3.654	DIST = 111.300	RUN OUT TANGENT
OFFTRACKING = 3.463	DIST = 113.300	RUN OUT TANGENT
OFFTRACKING = 3.282	DIST = 115.300	RUN OUT TANGENT
OFFTRACKING = 3.110	DIST = 117.300	RUN OUT TANGENT
OFFTRACKING = 2.947	DIST = 119.300	RUN OUT TANGENT
OFFTRACKING = 2.793	DIST = 121.300	RUN OUT TANGENT
OFFTRACKING = 2.647	DIST = 123.300	RUN OUT TANGENT
OFFTRACKING = 2.508	DIST = 125.300	RUN OUT TANGENT
OFFTRACKING = 2.377	DIST = 127.300	RUN OUT TANGENT
OFFTRACKING = 2.252	DIST = 129.300	RUN OUT TANGENT
OFFTRACKING = 2.134	DIST = 131.300	RUN OUT TANGENT
OFFTRACKING = 2.023	DIST = 133.300	RUN OUT TANGENT
OFFTRACKING = 1.917	DIST = 135.300	RUN OUT TANGENT
OFFTRACKING = 1.816	DIST = 137.300	RUN OUT TANGENT
OFFTRACKING = 1.721	DIST = 139.300	RUN OUT TANGENT
OFFTRACKING = 1.631	DIST = 141.300	RUN OUT TANGENT
OFFTRACKING = 1.546	DIST = 143.300	RUN OUT TANGENT
OFFTRACKING = 1.465	DIST = 145.300	RUN OUT TANGENT
OFFTRACKING = 1.388	DIST = 147.300	RUN OUT TANGENT
OFFTRACKING = 1.316	DIST = 149.300	RUN OUT TANGENT
OFFTRACKING = 1.247	DIST = 151.300	RUN OUT TANGENT
OFFTRACKING = 1.181	DIST = 153.300	RUN OUT TANGENT
OFFTRACKING = 1.120	DIST = 155.300	RUN OUT TANGENT
OFFTRACKING = 1.061	DIST = 157.300	RUN OUT TANGENT
OFFTRACKING = 1.005	DIST = 159.300	RUN OUT TANGENT
OFFTRACKING = .953	DIST = 161.300	RUN OUT TANGENT
OFFTRACKING = .903	DIST = 163.300	RUN OUT TANGENT
OFFTRACKING = .856	DIST = 165.300	RUN OUT TANGENT

Problem 4(b) C-TRAIN with A LOCKED STEERING B-DOLLY 14/12/83

OFFTRACKING = .811	DIST = 167.300	RUN OUT TANGENT
OFFTRACKING = .768	DIST = 169.300	RUN OUT TANGENT
OFFTRACKING = .728	DIST = 171.300	RUN OUT TANGENT
OFFTRACKING = .690	DIST = 173.300	RUN OUT TANGENT
OFFTRACKING = .654	DIST = 175.300	RUN OUT TANGENT
OFFTRACKING = .620	DIST = 177.300	RUN OUT TANGENT
OFFTRACKING = .587	DIST = 179.300	RUN OUT TANGENT
OFFTRACKING = .556	DIST = 181.300	RUN OUT TANGENT
OFFTRACKING = .527	DIST = 183.300	RUN OUT TANGENT
OFFTRACKING = .500	DIST = 185.300	RUN OUT TANGENT
OFFTRACKING = .474	DIST = 187.300	RUN OUT TANGENT
OFFTRACKING = .449	DIST = 189.300	RUN OUT TANGENT
OFFTRACKING = .425	DIST = 191.300	RUN OUT TANGENT
OFFTRACKING = .403	DIST = 193.300	RUN OUT TANGENT
OFFTRACKING = .382	DIST = 195.300	RUN OUT TANGENT
OFFTRACKING = .362	DIST = 197.300	RUN OUT TANGENT
OFFTRACKING = .343	DIST = 199.300	RUN OUT TANGENT
OFFTRACKING = .325	DIST = 201.300	RUN OUT TANGENT
OFFTRACKING = .308	DIST = 203.300	RUN OUT TANGENT
OFFTRACKING = .292	DIST = 205.300	RUN OUT TANGENT
OFFTRACKING = .277	DIST = 207.300	RUN OUT TANGENT
OFFTRACKING = .262	DIST = 209.300	RUN OUT TANGENT
OFFTRACKING = .248	DIST = 211.300	RUN OUT TANGENT
OFFTRACKING = .235	DIST = 213.300	RUN OUT TANGENT
OFFTRACKING = .223	DIST = 215.300	RUN OUT TANGENT

THE OFFTRACKING VALUES LISTED ARE DEFINED AS THE LATERAL SHIFT OF THE LAST AXLE GROUP OF THE REAR TRAILER WITH RESPECT TO THE PATH FOLLOWED BY THE STEERING AXLE OF THE TRACTOR. ALL POINTS ARE MEASURED FROM THE CENTER LINE OF THE VEHICLE AXIS.

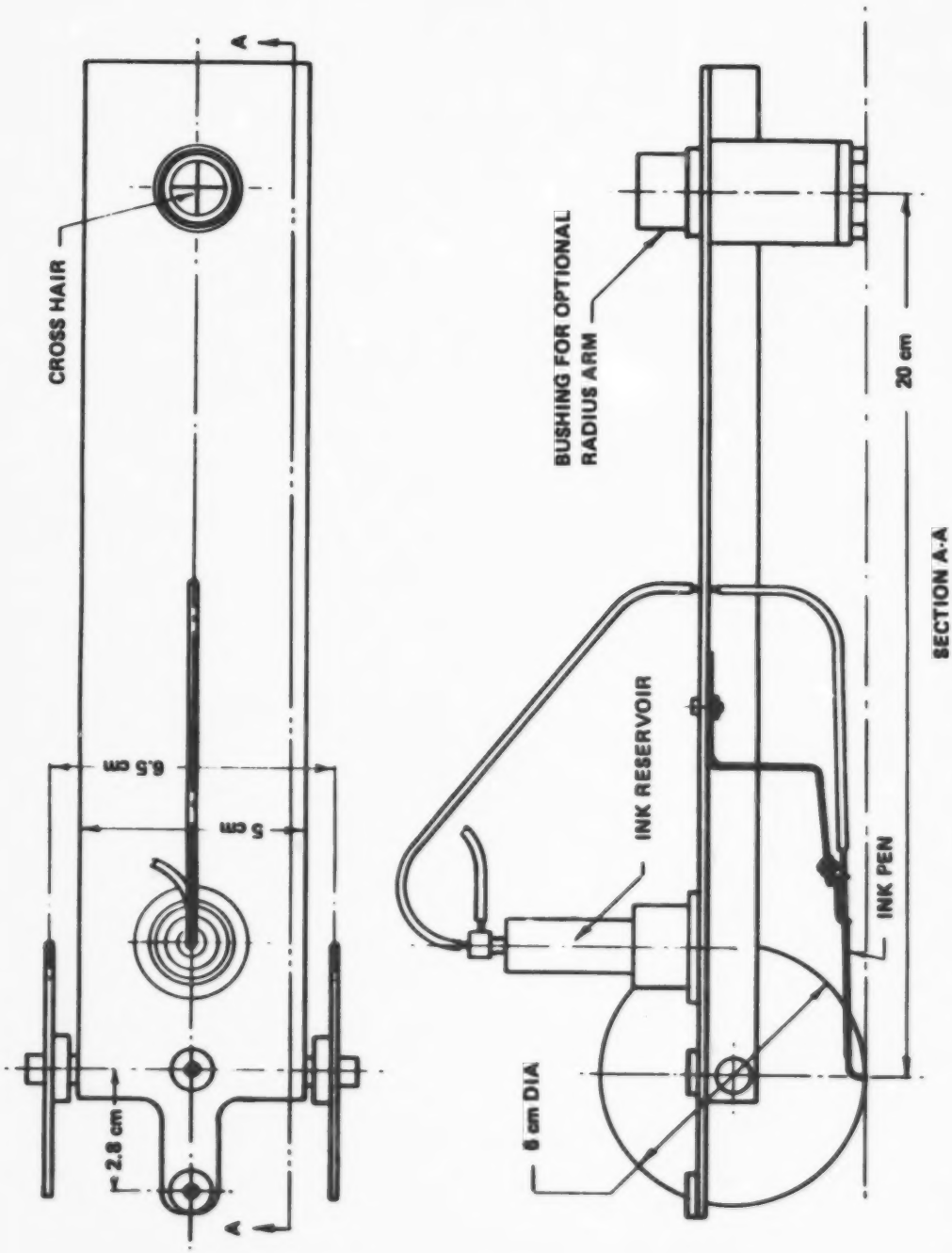
DIST IS THE POINT ON THE PATH WHERE THE GIVEN OFFTRACK VALUE APPLIES. DIST=0 REFERS TO THE START OF THE CURVE. THEREFORE THE LEAD TANGENT PORTION IS NEGATIVE.

ANG DISPL IS THE ANGULAR DISPLACEMENT OF THE CONSTANT RADIUS CURVE WHERE ZERO IS AT THE BEGINNING OF THE CURVE.

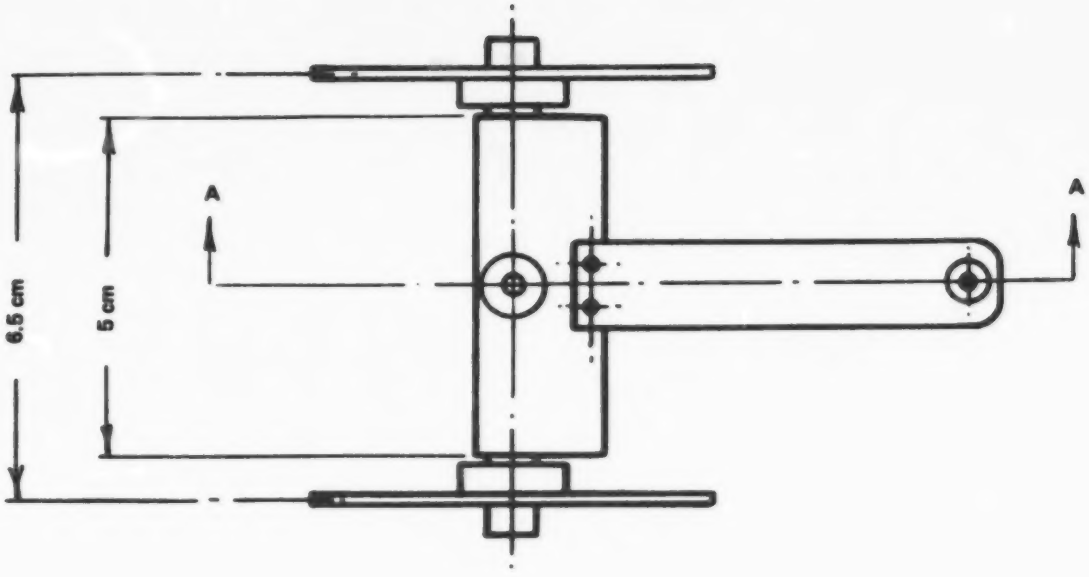
********** All units are in FEET **********

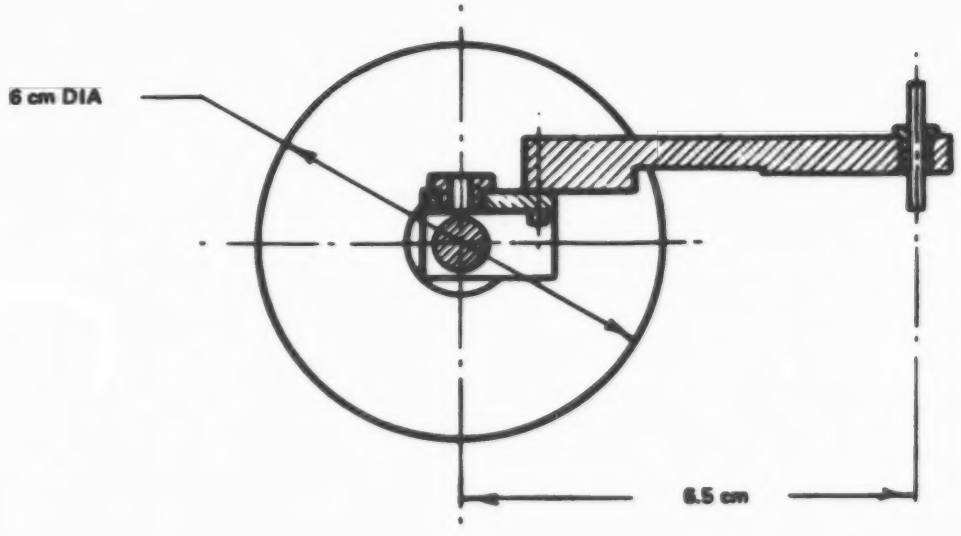
APPENDIX B

PHYSICAL OFF TRACKING MODEL



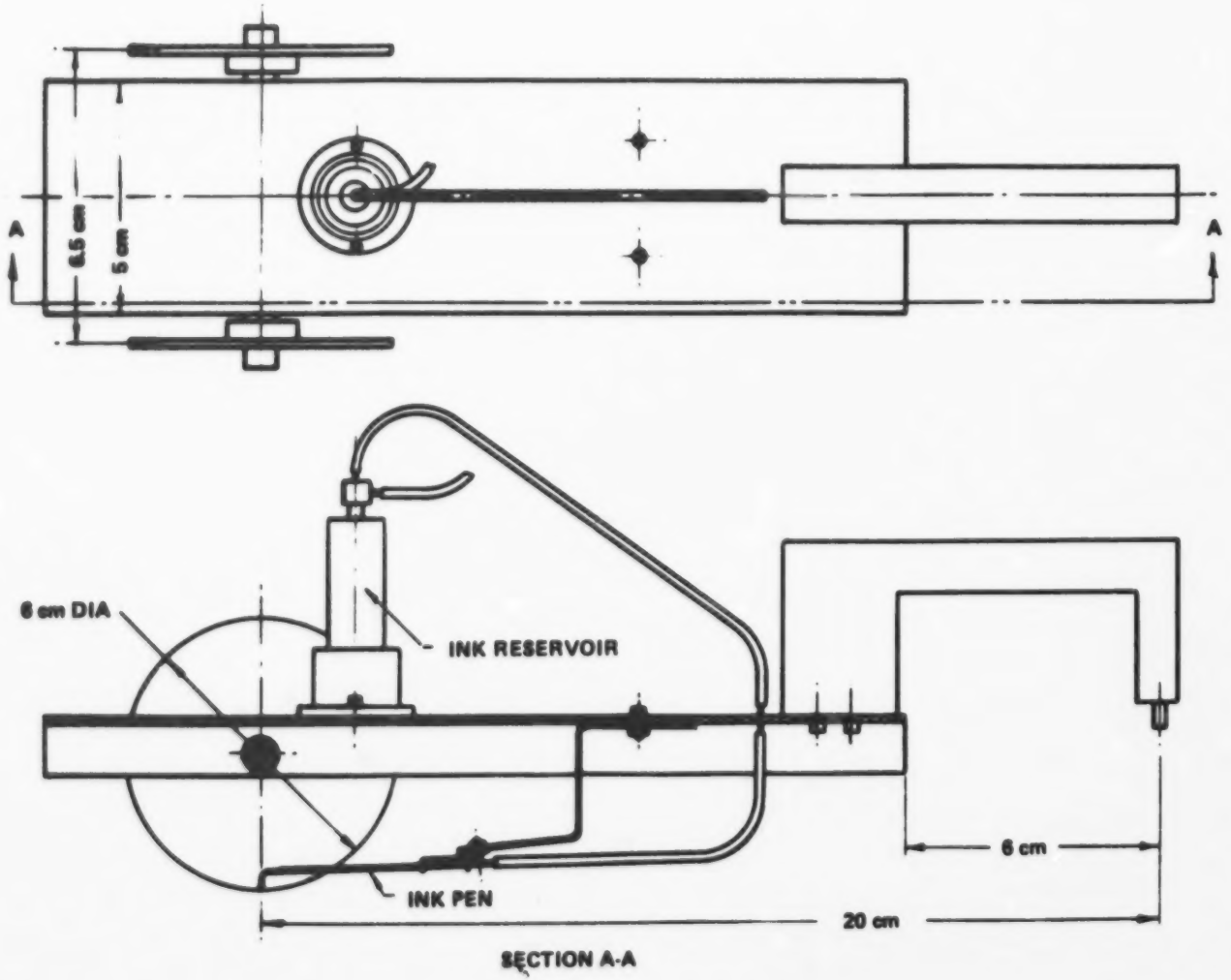
PLAN AND SIDE VIEW OF LEAD UNIT OF SCALE MODEL





SECTION A-A

PLAN AND SIDE VIEW OF A-DOLLY SCALE MODEL



PLAN AND SIDE VIEW OF TRAILING UNIT SCALE MODEL

APPENDIX C

VALIDATION GRAPHS AND EXPLANATION

The following collection of plots, compared data generated by the piecewise solution with data from physical model tests and full scale field experiments. The first series, Figures C1 to C6 demonstrate the agreement of the mathematical model with data from scale model experiments. Within this series curve durations of 540°, 90° and 45° are shown. The vehicle combinations used were a single unit, double unit A- and B-trains and a triple B-train.

The second series, Figures C7 to C12 demonstrate the agreement of the mathematical model with data from full size track trailer experiments. Within this series, curve duration of 360°, 180°, 90° and 45° are shown. The vehicle combinations included in the valication are a tractor semi trailer; a double trailer A-train; a double trailer C-train with a self-steering dolly axle (the C-train is a multiple unit vehicle where double draw bar dollies i.e. B-dollies, are used in place of the conventional A-dolly); and a double C-train with a non-steering B-dolly axle (a vehicle with a non-steering B-dolly approximately a B-train). Note that asterisk points are the measured data and the continuous lines are the analytical results.

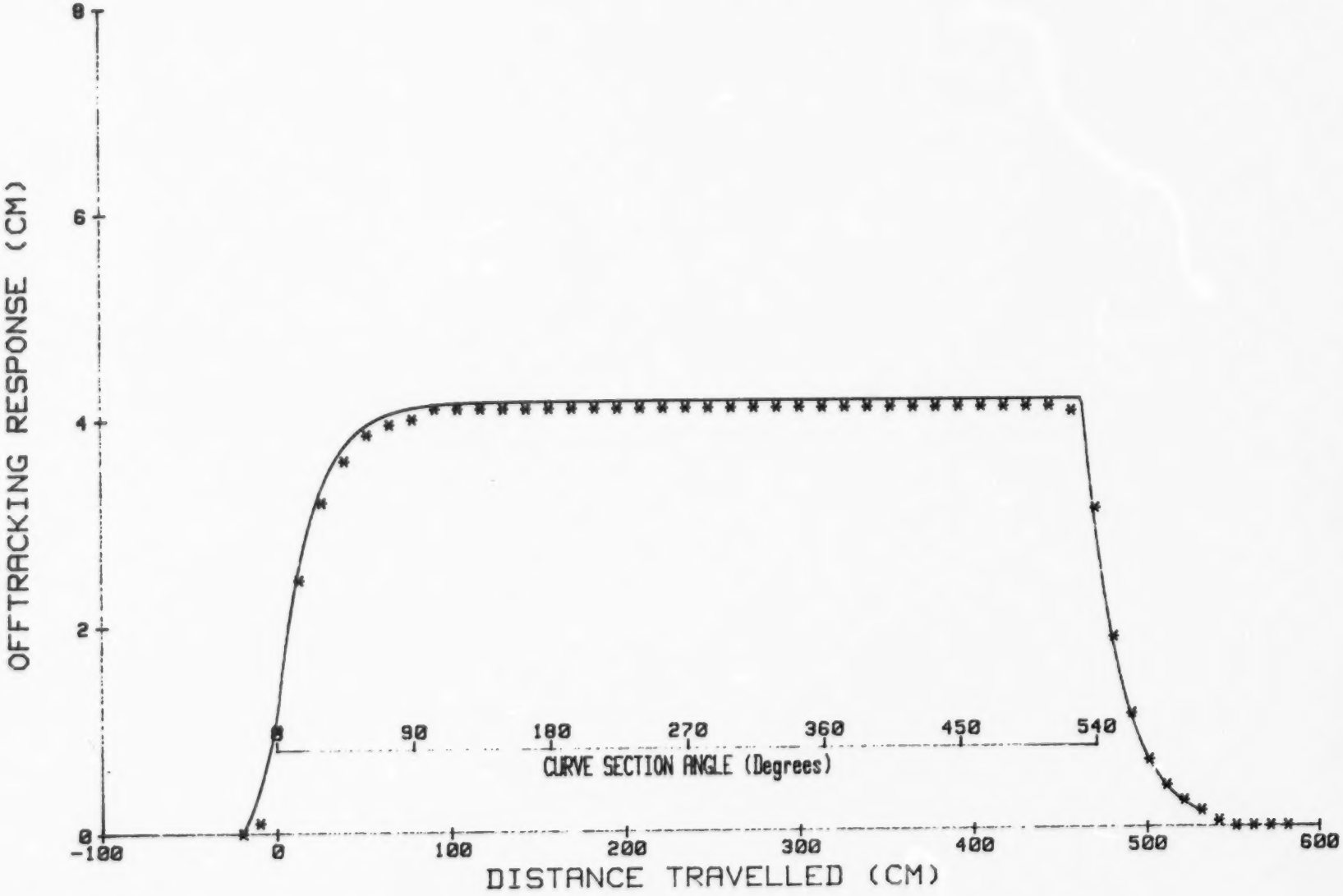


FIG. C1: MODEL TEST – SINGLE UNIT 50 cm RADIUS 540 DEG. TURN

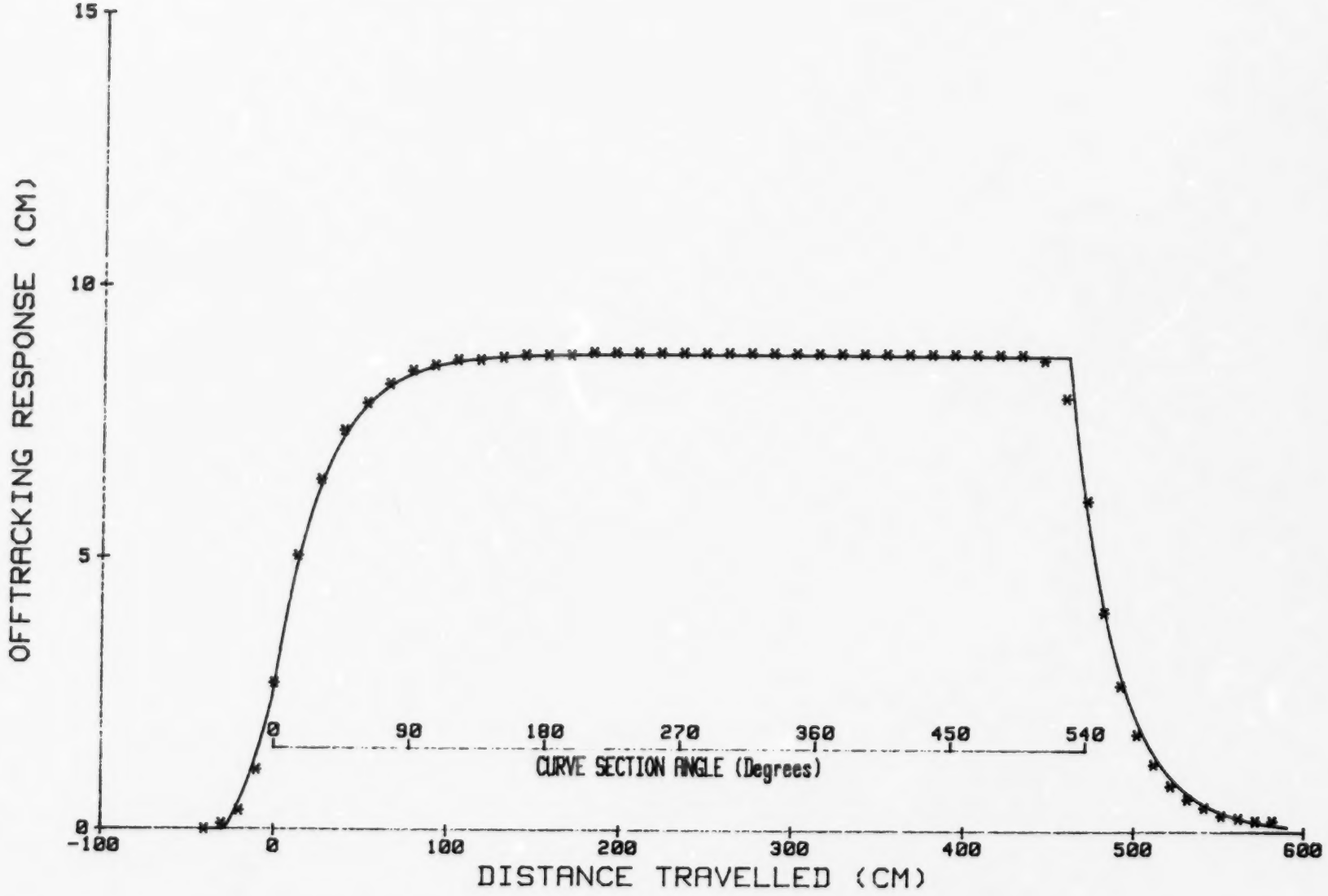


FIG. C2: MODEL TEST – DOUBLE UNIT B-TRAIN 50 cm RADIUS 540 DEG. TURN

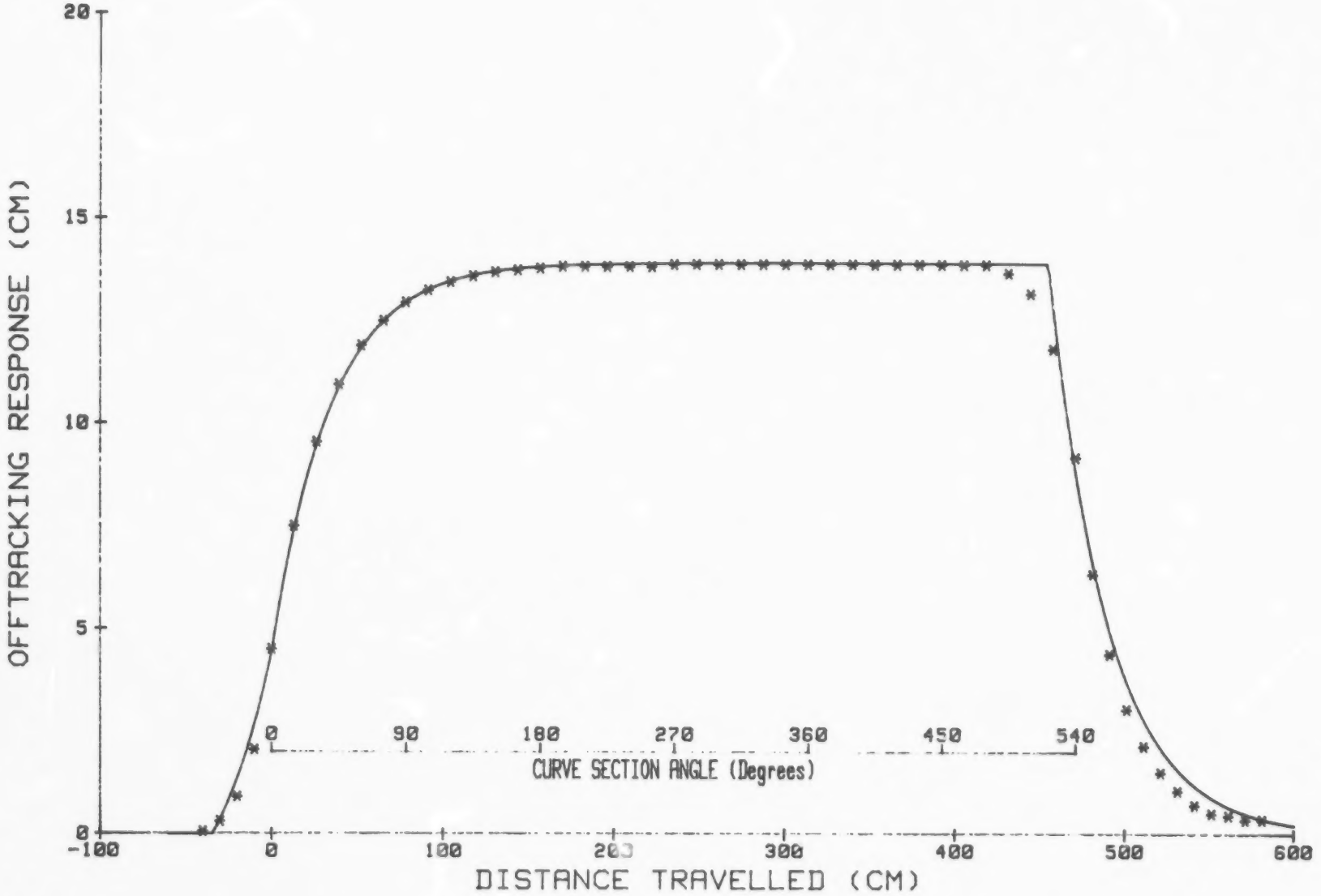


FIG. C3: MODEL TEST – TRIPLE UNIT B-TRAIN 50 cm RADIUS 540 DEG. TURN

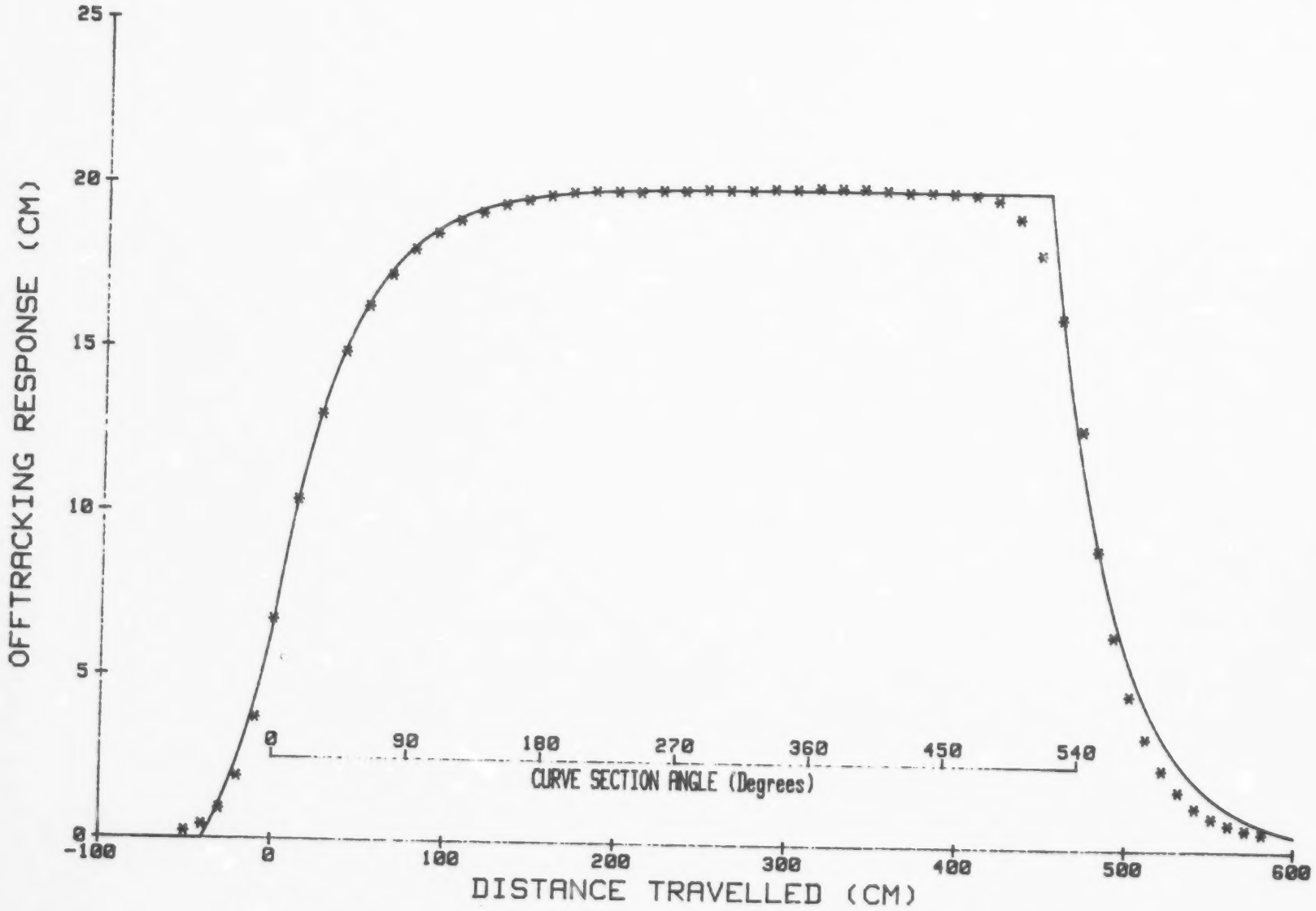


FIG. C4: MODEL TEST – FOUR UNIT B-TRAIN 50.0 cm RADIUS 540 DEG. TURN

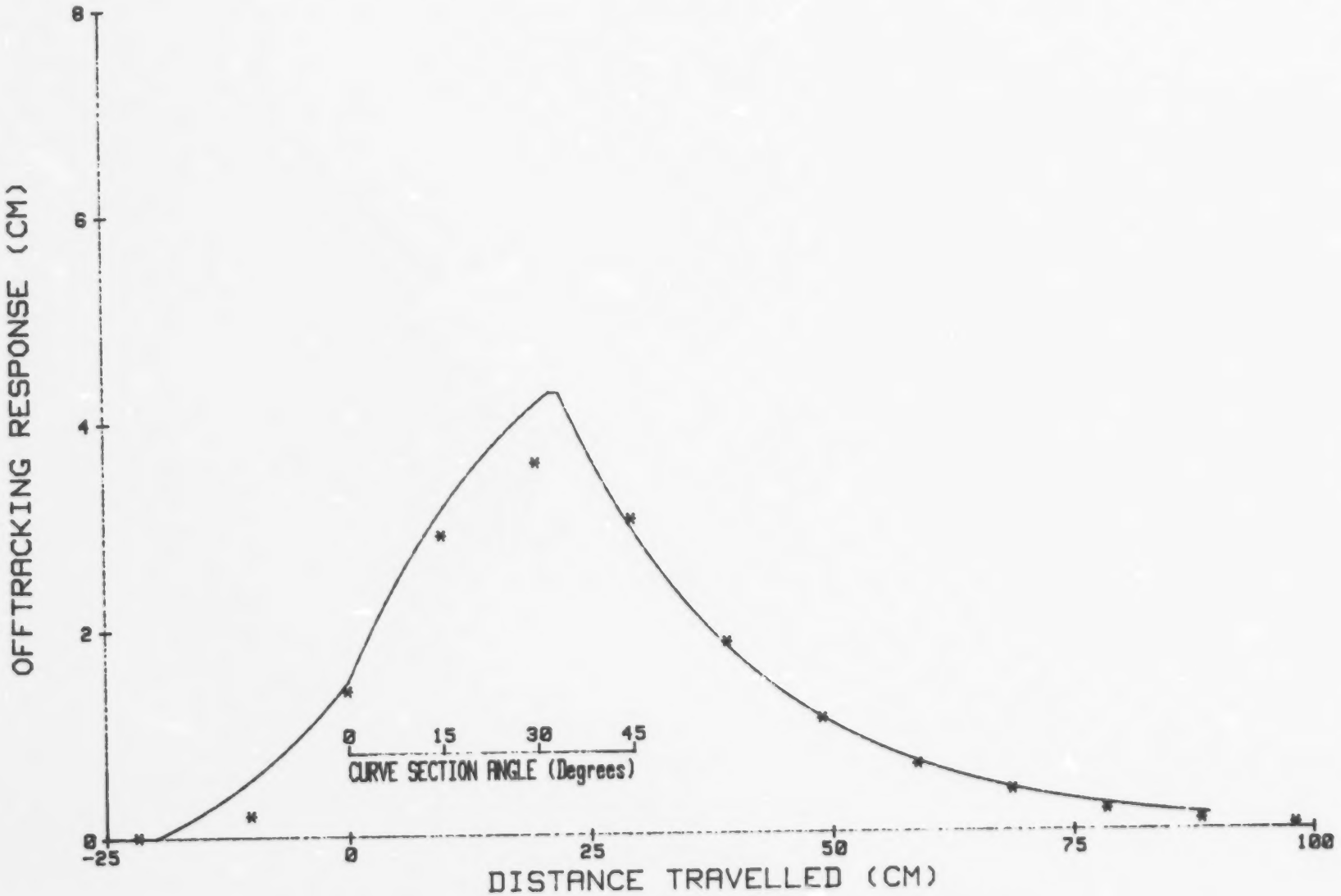


FIG. C5: MODEL TEST – SINGLE UNIT 37.5 cm RADIUS 45 DEG. TURN

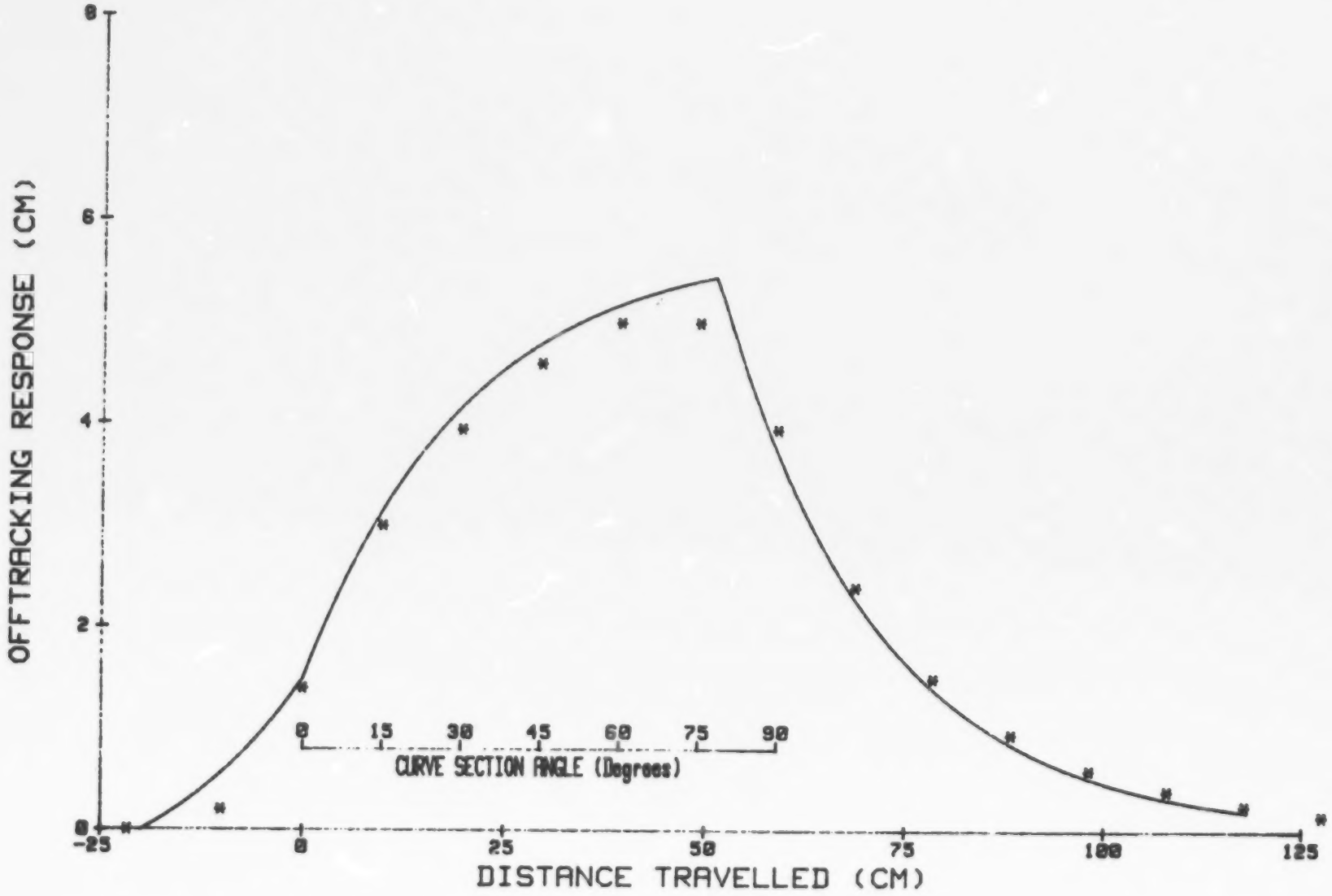


FIG. C8: MODEL TEST – SINGLE UNIT 37.5 cm RADIUS 90 DEG. TURN

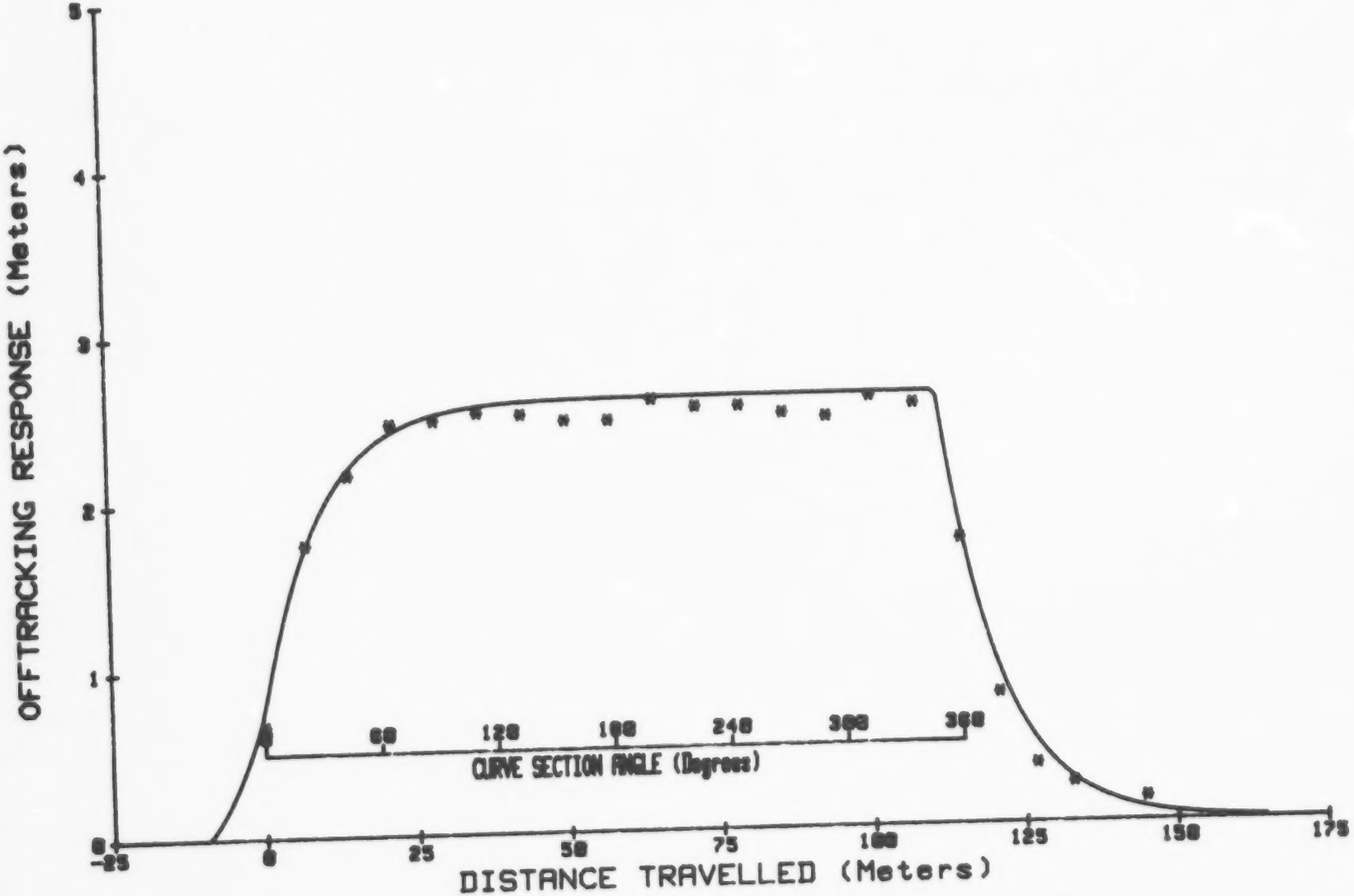


FIG. C7: FIELD TEST – SEMI TRAILER 18.3 m RADIUS 360 DEG. TURN

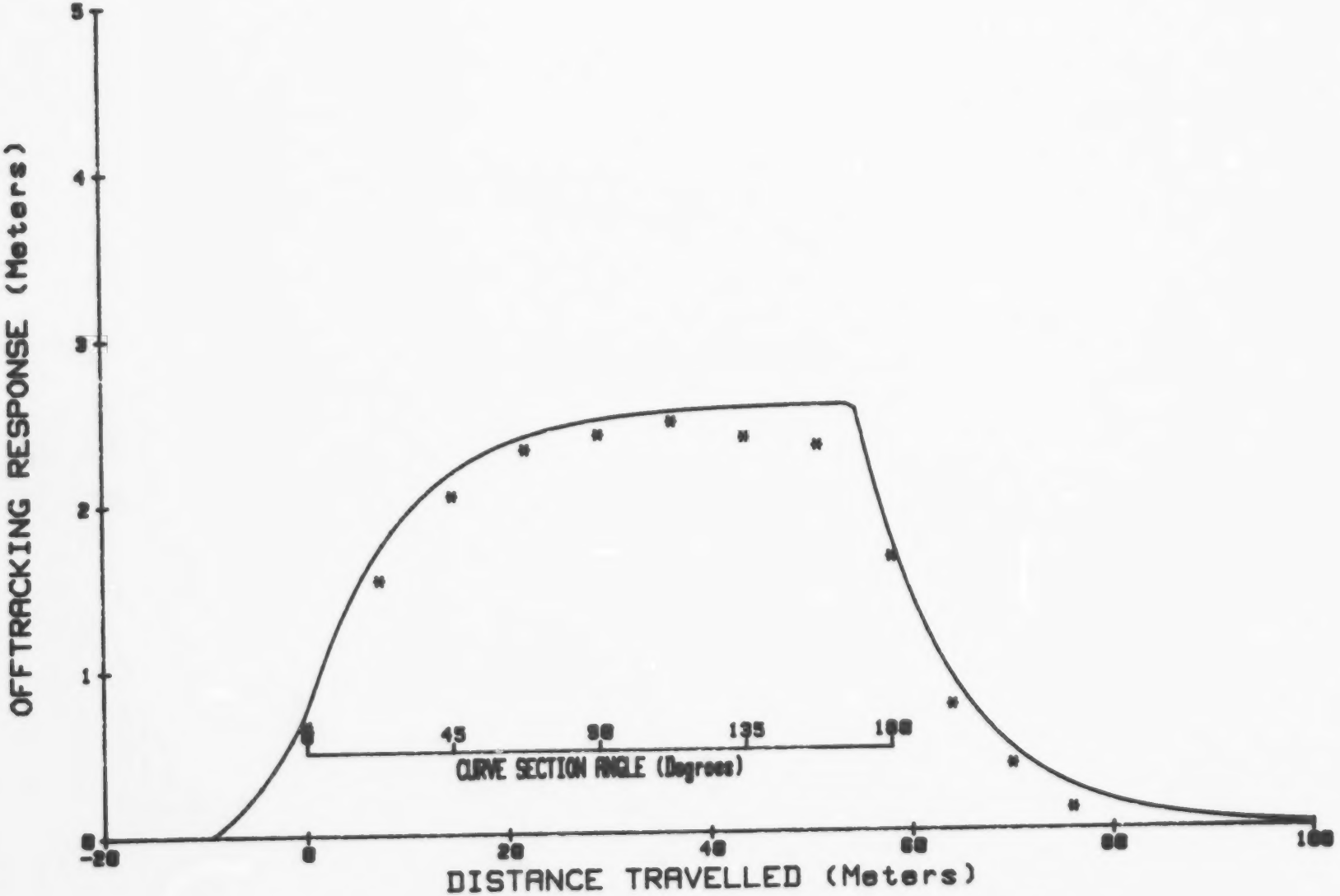


FIG. C8: FIELD TEST – SEMI TRAILER 18.5 m RADIUS 180 DEG. TURN

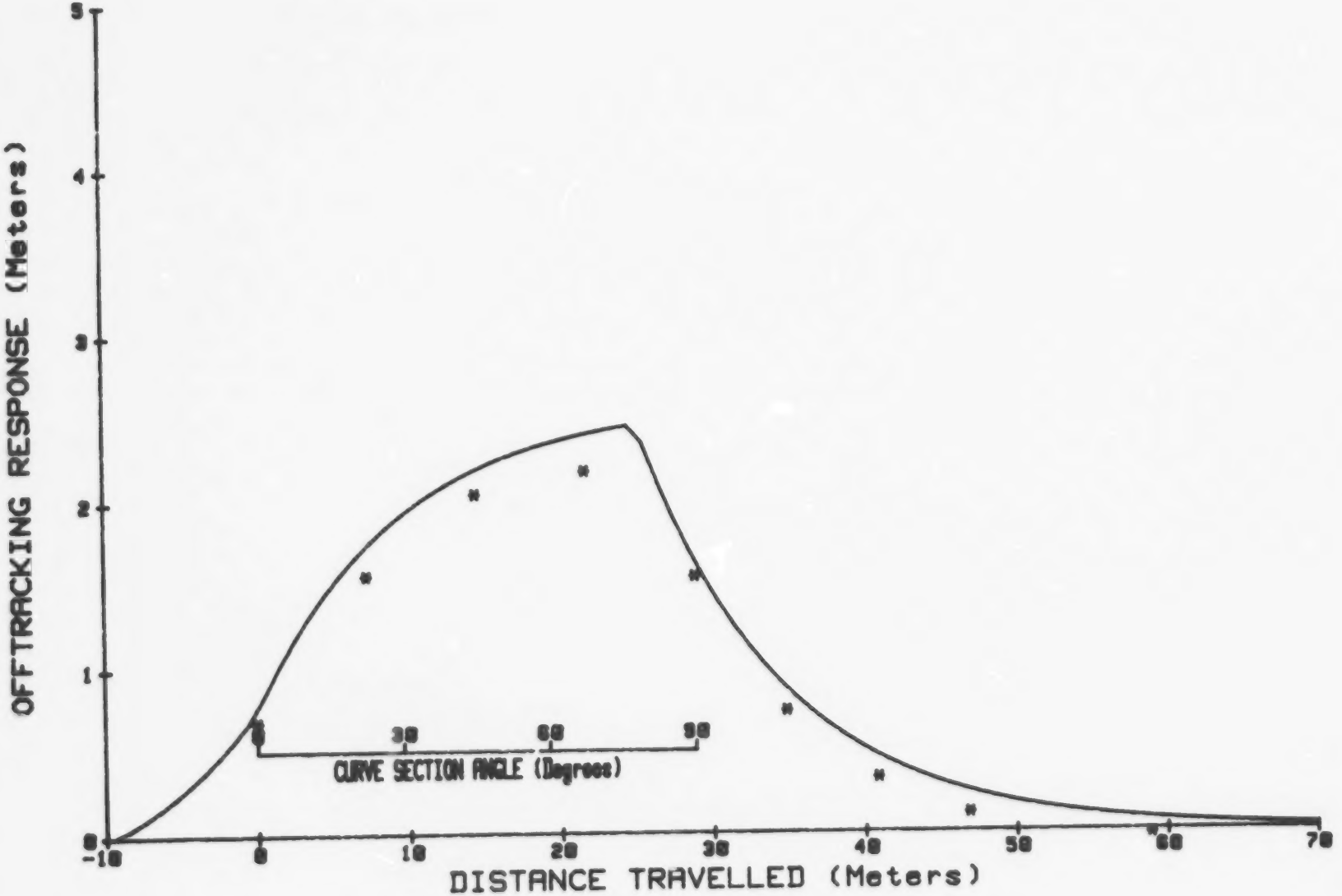


FIG. C9: FIELD TEST – SEMI TRAILER 18.4 m RADIUS 90 DEG. TURN

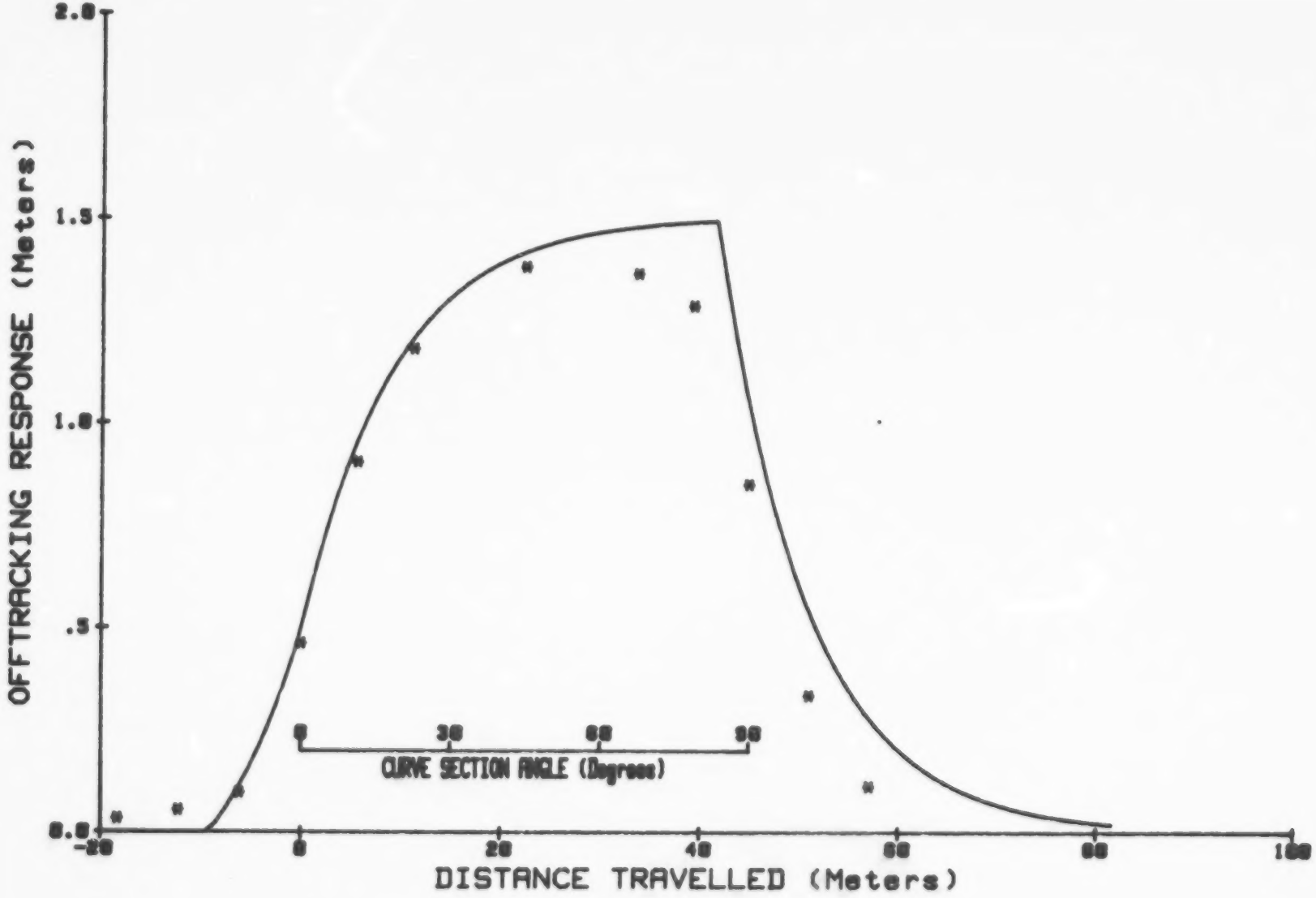


FIG. C10: FIELD TEST – DOUBLE C-TRAIN (DOLLY STEER LOCKED) 28.6 m RADIUS

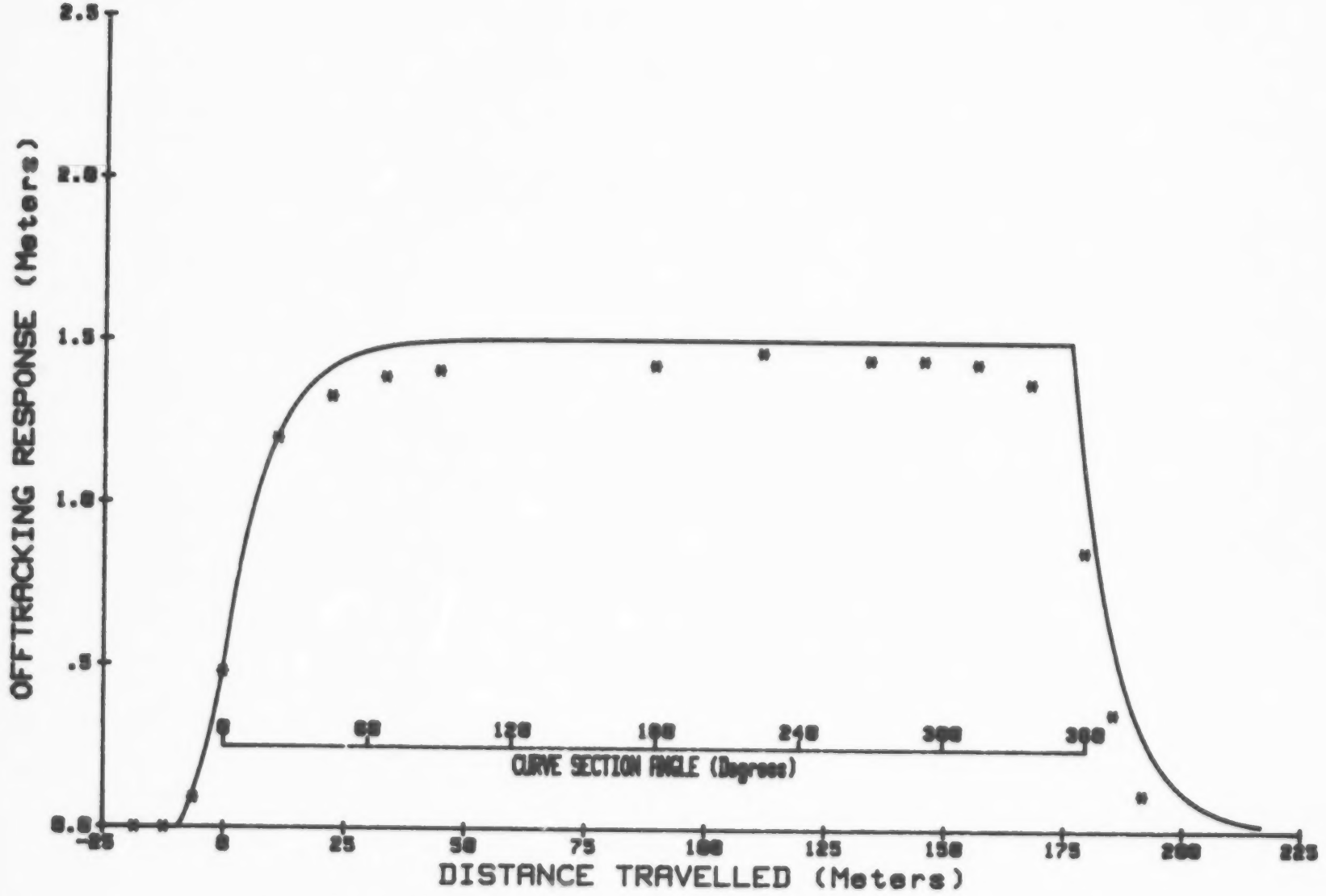


FIG. C11: FIELD TEST – DOUBLE C-TRAIN (DOLLY STEEL LOCKED) 28.6 m RADIUS

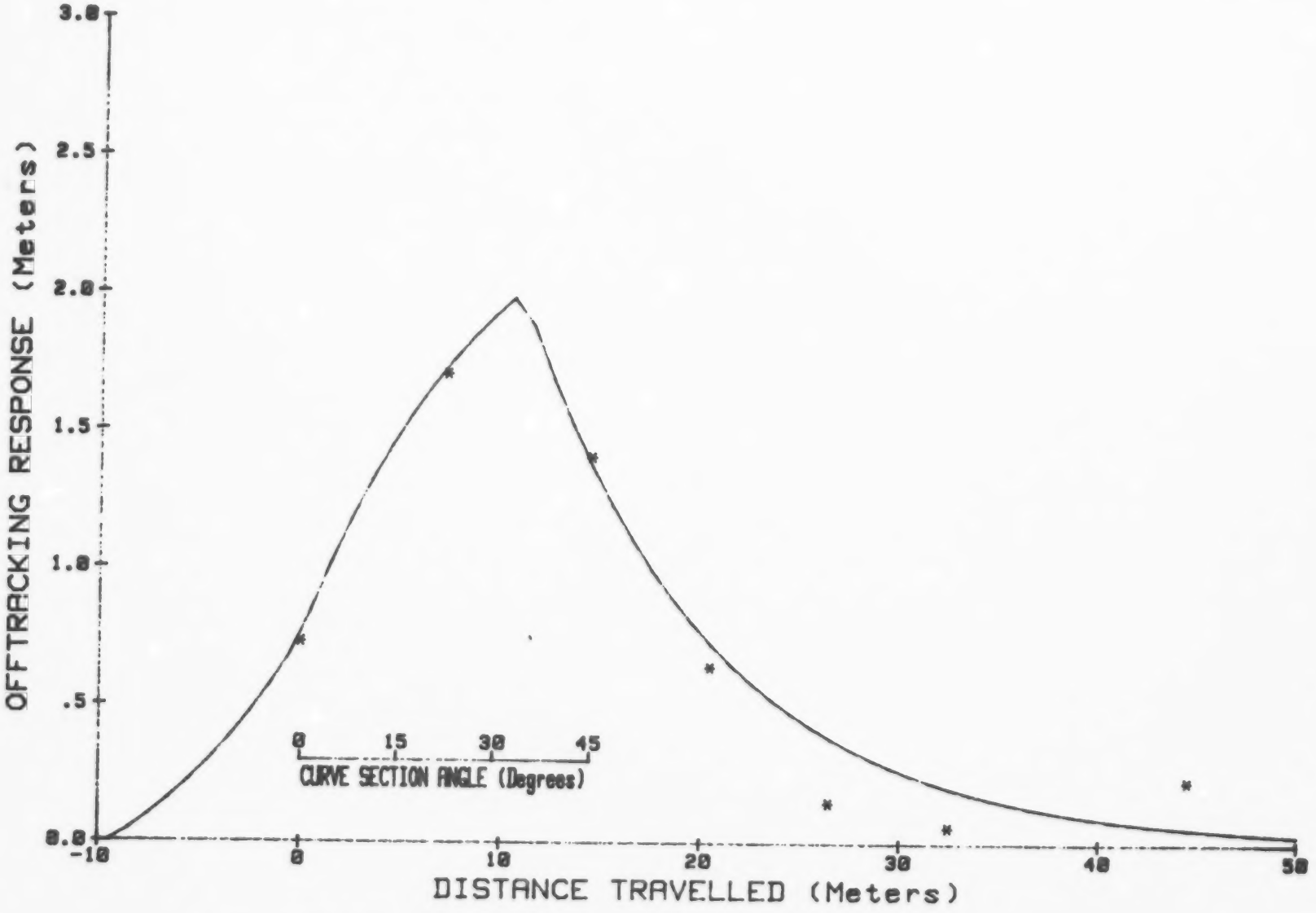


FIG. C12: FIELD TEST – SEMI TRAILER 18.4 m RADIUS 45 DEG. TURN

APPENDIX D

'BASIC' PROGRAM FOR TRANSIENT OFF TRACKING SUITABLE FOR HEWLETT PACKARD 9835A DESK TOP COMPUTER

```
10    ! OFTRK1       OFF-TRACKING PROGRAM                RONN SENN     28/4/83
20    !
30    PRINTER IS 16
40    PRINT PAGE
50    PRINT "THIS PROGRAM CALCULATES TRANSIENT OFF-TRACKING OF MULTI-UNIT"
60    PRINT "            VEHICLES WITH OR WITHOUT DOLLIES."
70    PRINT LIN(3)
80    PRINT "HARDCOPY: - A.S$11 inch paper"
90    PRINT "          - 8 lines per inch spacing"
100   PRINT "          - paper to be at TOF before starting program"
110   !
120   DIM T$[80],Title$[60]
130   IF Ans$="Y" THEN 2030
140   INPUT "Date?   (D/M/Y)",D$
150   DISP "Title?";TAB(61);"(60 char.";
160   INPUT Title$
170   T$=Title$&"    "&D$
180   INPUT "ENTER: Curve Duration (Angular Displacement) in DEGREES",Cd
190   Th=2*PI/(360/Cd)
200   INPUT "UNITS in Feet or Meters?   (F/M)",Um$
210   IF Um$="F" THEN Um$="Ft"
220   IF Um$="Ft" THEN U$="Feet"
230   IF Um$="M" THEN U$="Meters"
240   DISP "ENTER: Tractor Wheelbase in ";U$;
250   INPUT A
260   DISP "ENTER: Tractor 5th Wheel Position (Forward Positive) in ";U$;
270   INPUT A1
280   DISP "ENTER: FIRST Trailer Wheelbase in ";U$;
290   INPUT B
300   DISP "ENTER: FIRST Trailer Rear Overhang in ";U$;
310   INPUT B1
320   DISP "ENTER: FIRST Dolly Hitch Length in ";U$;
330   INPUT B2
340   DISP "ENTER: SECOND Trailer Wheelbase in ";U$;
350   INPUT C
360   DISP "ENTER: SECOND Trailer Overhang in ";U$;
370   INPUT C1
380   DISP "ENTER: SECOND Dolly Hitch Length in ";U$;
390   INPUT C2
400   DISP "ENTER: THIRD Trailer Wheelbase in ";U$;
410   INPUT D
420   DISP "ENTER: THIRD Trailer Overhang in ";U$;
430   INPUT D1
440   DISP "ENTER: THIRD Dolly Hitch Length in ";U$;
450   INPUT D2
460   DISP "ENTER: FOURTH Trailer Wheelbase in ";U$;
470   INPUT E
480   DISP "ENTER: Mean Curve Radius at Tractor Front Axle Center in ";U$;
490   INPUT R
500   PRINT PAGE
510   ! ************************************************************************
520   Wbi=A+B+B2+C+C2+D+D1+D2+E
530   Wbi2=A^2+B^2+B2^2+C^2+C2^2+D^2+D2^2+E^2
540   Dmj=-A1+B1+C1+D1
```

```
550  Dmj2=A1^2+B1^2+C1^2+D1^2
560  Wbe=(Whi2-Dmj2)^.5
570  IF Whe>R THEN 650
580  Ots=R-(R^2-Whi2+Dmj2)^.5
590  Wht=Whi+Dmj
600  Z=.6*Whe*Wht/(Wbe+Wbt)
610  Cc=Whe*LOG((Whe^2+2*R*Ots-Ots^2)*EXP(-1)/(2*R*Ots))+Th*R
620  Otm=Ots-Ots*EXP(-(Cc+Z)/Wbe)
630  S=-Wht
640  GOTO 700
650  ' ************************************************************************
***
660  PRINT "INVALID SOLUTION",LIN(1)
670  PRINT "Effective wheelbase exceeds curve radius."
680  PRINT "CHOOSE new radius larger than ";Whe;Um$
690  GOTO 480
700  ' ************************************************************************
***
710  INPUT "Do you want a hard copy?   (Y/N)",Ahc$
720  IF Ahc$="Y" THEN PRINTER IS 7,1
730  PRINT
740  PRINT "            ********************************************"
750  PRINT "            ********************************************"
760  PRINT "            ************    OFTRK1      ***************"
770  PRINT "            ********************************************"
780  PRINT "            ********************J. WOODROOFFE & R. SENN"
790  PRINT
800  PRINT
810  PRINT
820  PRINT T$
830  PRINT
840  PRINT
850  PRINT "************************************************************************
*********"
860  FIXED 2
870  PRINT
880  PRINT "           ROAD AND VEHICLE SPECIFICATIONS"
890  PRINT
900  PRINT "CURVE DURATION =";Cd;" Degrees (";Th;" Radians)"
910  PRINT "CURVE RADIUS =";R;Um$
920  PRINT
930  PRINT "TRACTOR WHEELBASE=";A;Um$;TAB(36);"TRACTOR FIFTH WHEEL POSITION
=";A1;Um$
940  PRINT "FIRST TRAILER WHEELBASE=";B;Um$;TAB(36);"FIRST TRAILER OVERHANG
=";B1;Um$
950  PRINT "FIRST DOLLY HITCH LENGTH=";B2;Um$;TAB(36);"SECOND TRAILER WHEEL
BASE=";C;Um$
960  PRINT "SECOND TRAILER OVERHANG=";C1;Um$;TAB(36);"SECOND DOLLY HITCH LE
NGTH=";C2;Um$
970  PRINT "THIRD TRAILER WHEELBASE=";D;Um$;TAB(36);"THIRD TRAILER OVERHANG
=";D1;Um$
980  PRINT "THIRD DOLLY HITCH LENGTH=";D2;Um$;TAB(36);"FOURTH TRAILER WHEEL
BASE";E;Um$
990  PRINT
```

```
1000 PRINT "**********************************************************************
**********"
1010 PRINT
1020 FIXED 2
1030 PRINT "STEADY STATE OFF TRACKING =";Ots;Um$;"  TOTAL WHEEL BASE =";Wht
;Um$
1040 PRINT "EFFECTIVE WHEEL BASE =";Whe;Um$
1050 PRINT
1060 PRINT "MAXIMUM OFF-TRACKING =";Otm;Um$;" occurs ";Cc;Um$;" at";Cc*360/
(2*PI*R);"Degrees"
1070 PRINT "from the start of the curve."
1080 PRINT
1090 PRINT "^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
^^^^^^^^^^"
1100 GOSUB Pause
1110 FIXED 3
1120 Lines=33
1130 Otso=Ots-Ots*EXP(-7/Whe)     ! OFFTRACK AT S=0 USED TO CALCULATE
1140                              ! SLOPE FOR LEAD IN TANGENT PORTION
1150 Startloop:Lines=Lines+1
1160 Ot2=Ots-Ots*EXP((-S-7)/Whe)
1170 Ot3=Otm*EXP((-S+Cc)/Whe)
1180 IF (S<Cc) OR (S=Cc) THEN 1200
1190 GOTO 1360
1200 IF (S>0) OR (S=0) THEN 1320
1210 IF S<-Whe THEN 1230
1220 GOTO 1270
1230 PRINT " OFFTRACKING = 000";TAB(26);"DIST =";S;TAB(43);"LEAD IN TANGENT
"
1240 Ot=0
1250 Ad=0
1260 GOTO 1440
1270 Ot1=-.582*Otso+1.582*Otso*EXP(S/Whe)     ! OFFTRACK LEAD IN EXP
1280 PRINT " OFFTRACKING =";Ot1;TAB(26);"DIST =";S;TAB(43);"LEAD IN TANGENT
 EXP FUNC"
1290 Ot=Ot1
1300 Ad=0
1310 GOTO 1440
1320 PRINT " OFFTRACKING =";Ot2;TAB(26);"DIST =";S;TAB(43);"ANG DISPL =";S*
360/(2*PI*R);"DEG"
1330 Ot=Ot2
1340 Ad=S*360/(2*PI*R)
1350 GOTO 1440
1360 IF S/R>Th THEN 1410
1370 PRINT " OFFTRACKING =";Ot3;TAB(26);"DIST =";S;TAB(43);"ANG DISPL =";S*
360/(2*PI*R);"DEG"
1380 Ot=Ot3
1390 Ad=S*360/(2*PI*R)
1400 GOTO 1440
1410 PRINT " OFFTRACKING =";Ot3;TAB(26);"DIST =";S;TAB(43);"RUN OUT TANGENT
"
1420 Ot=Ot3
1430 Ad=0
1440 S=S+2
```

```
1450 IF (Lines(79) OR (Ahc$="N") THEN 1490
1460 PRINT PAGE
1470 PRINT LIN(1),T$,LIN(2)
1480 Lines=4
1490 EndLoop:IF S(2*PI*R*Cd/360+2*Wht THEN 1150
1500 IF (79-Lines)18) OR (Ahc$="N") THEN 1550
1510 PRINT PAGE
1520 PRINT T$,LIN(2)
1530 PRINT "         ************************************************"
1540 PRINT "         ************************************************"
1550 PRINT
1560 PRINT " THE OFFTRACKING VALUES LISTED ARE DEFINED AS THE LATERAL"
1570 PRINT " SHIFT OF THE LAST AXLE GROUP OF THE REAR TRAILER WITH"
1580 PRINT " RESPECT TO THE PATH FOLLOWED BY THE STEERING AXLE OF THE"
1590 PRINT " TRACTOR. ALL POINTS ARE MEASURED FROM THE CENTER LINE OF"
1600 PRINT " THE VEHICLE AXIS."
1610 PRINT ""
1620 PRINT " DIST IS THE POINT ON THE PATH WHERE THE GIVEN OFFTRACK VALUE"
1630 PRINT " APPLIES. DIST=0 REFERS TO THE START OF THE CURVE. THEREFORE"
1640 PRINT " THE LEAD TANGENT PORTION IS NEGATIVE."
1650 PRINT ""
1660 PRINT " ANG DISPL IS THE ANGULAR DISPLACEMENT OF THE CONSTANT RADIUS"
1670 PRINT " CURVE WHERE ZERO IS AT THE BEGINNING OF THE CURVE."
1680 PRINT ""
1690 IF Um$="Ft" THEN PRINT "     ***********  All units are in FEET  ******
*********"
1700 IF Um$="M" THEN PRINT "     ***********  All units are in METERS  *****
**********"
1710 IF Ahc$="Y" THEN PRINT PAGE
1720 PRINTER IS 16
1730 End: DISP ""
1740      PRINT "END OF PROGRAM"
1750 STOP
1760 Pause: IF Ahc$="Y" THEN 1800
1770 DISP "Push CONTINUE when ready."
1780 PAUSE
1790 DISP ""
1800 RETURN
1810 END
```

APPENDIX E

DERIVATION OF Z

A first approximation of z is found from the following geometric construction.

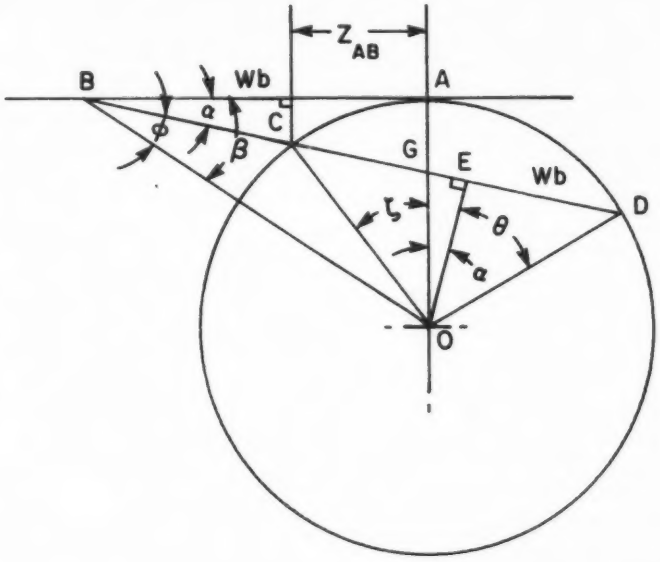


FIG. E-1: CONSTRUCTION FOR DERIVATION OF Z

Logic

$$AB = CD$$

$$\Delta OAB \rightarrow \measuredangle \beta, OB$$

$$\Delta OCE \rightarrow OE$$

$$\Delta OBE \rightarrow \measuredangle \phi$$

$$\measuredangle \beta - \measuredangle \phi = \measuredangle \alpha$$

$$\measuredangle \alpha \rightarrow AG$$

$$\beta = \tan^{-1} \frac{R}{wb}$$

$$OB = (R^2 + wb^2)^{1/2}$$

$$OE = \left(R^2 \frac{wb^2}{4}\right)^{1/2}$$

$$\phi = \sin^{-1} \left[\frac{\left(R^2 \frac{w_b^2}{4}\right)^2}{(wb^2 + r^2)^{1/2}} \right]$$

$$\alpha = \beta - \phi$$

$$\alpha = \tan^{-1} \frac{R}{wb} - \sin^{-1} \left[\frac{\left(R^2 \frac{wb^2}{4}\right)^{1/2}}{(wb^2 + R^2)^{1/2}} \right]$$

$$AG = wb \tan \alpha \qquad \text{1-E}$$

Also

$$\xi = \theta - \alpha$$

$$\theta = \sin^{-1} \frac{wb/2}{R}$$

$$Z_{AB} = R \sin(\theta - \alpha) \qquad \text{2-E}$$

It is clear that the trailing axle of a vehicle will not trace a straight line B through C as illustrated in Figure E1. The actual trace will be a curved line which lies between the straight line BC and the following exponential trace OT, derived below.

Using the same form of equation derived for the entry tangent, the off tracking at Z_{OT} is

$$OT_Z = AG - \left[\frac{AG}{0.632} - \frac{AG}{0.632} \exp - \frac{Z_{OT}}{wb} \right]$$

rearranging the equations to solve for Z_{OT}

$$\exp\left(\frac{Z_{OT}}{wb}\right) = \frac{.632}{AG}\left(OT_Z - AG + \frac{AG}{.6321}\right)$$

$$Z_{OT} = -wb\,\ell n\left[\frac{.632}{AG}\left(OT_Z - AG + \frac{AG}{.632}\right)\right] \qquad \text{3-E}$$

To solve for Z_{OT}, a second equation based upon the definition of a circle is used.

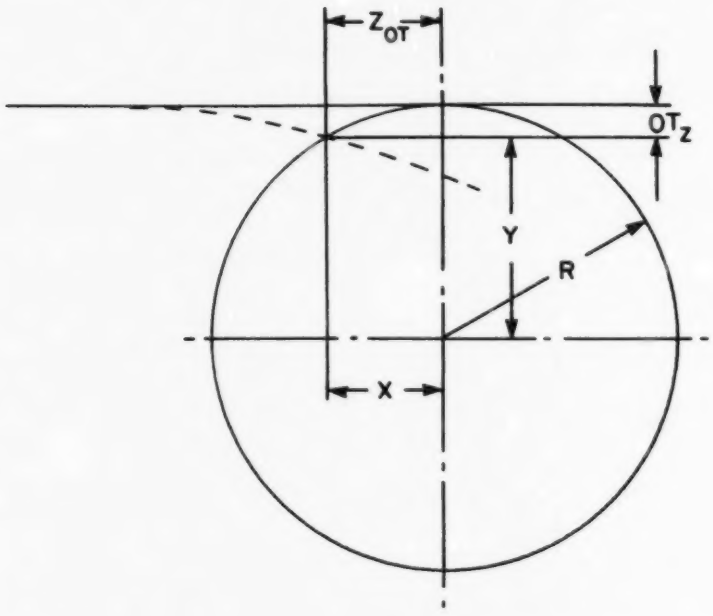


FIG. E-3

$$Y = R - OT_Z$$

$$R^2 = x^2 + (R - OT_Z)^2$$

since $x = Z_{OT}$

$$Z_{OT}^{\ 2} = R^2 - (R - OT_Z)^2 \qquad \text{ZO}$$

$$Z_{OT} = 2ROT_Z - OT_Z^2\ ^{1/2} \qquad \text{4-E}$$

Z_{OT} can be found by solving Equations 3-E and 4-E by iteration.

Since the true value of Z lies between Z_{AB} (given by Eq. 2-E) and Z_{OT} the best practical approximation of Z if found by taking the average of Z_{AB} and Z_{OT}.

Calculations of Z have been made over a wide range of wheelbase (wb) and curve radius R. The resultant value of Z all fall very close to 0.3 wb.

The following table shows the values of Z_{AB}, Z_{OT} and Z for a range of wheelbases and curve radius.

Wheelbase	10	15	20	25	30
3	Z_{AB} 1.147 Z_{OT} 0.652 Z 0.900	Z_{AB} 1.146 Z_{OT} 0.616 Z 0.881	Z_{AB} 1.146 Z_{OT} 0.679 Z 0.913	Z_{AB} 1.146 Z_{OT} 0.629 Z 0.890	Z_{AB} 1.146 Z_{OT} 0.654 Z 0.900
	Z = 0.300 wb	Z = 0.294 wb	Z = 0.304 wb	Z = 0.297 wb	Z = 0.300 wb
6	Z_{AB} 2.302 Z_{OT} 1.302 Z 1.802	Z_{AB} 2.296 Z_{OT} 1.282 Z 1.789	Z_{AB} 2.294 Z_{OT} 1.279 Z 1.787	Z_{AB} 2.293 Z_{OT} 1.287 Z 1.790	Z_{AB} 2.293 Z_{OT} 1.269 Z 1.781
	Z = 0.300 wb	Z = 0.298 wb	Z = 0.298 wb	Z = 0.298 wb	Z = 0.292 wb
9	Z_{AB} 3.473 Z_{OT} 2.000 Z 2.737	Z_{AB} 3.452 Z_{OT} 1.947 Z 2.699	Z_{AB} 3.446 Z_{OT} 1.939 Z 2.692	Z_{AB} 3.443 Z_{OT} 1.920 Z 2.681	Z_{AB} 3.441 Z_{OT} 1.906 Z 2.674
	Z = 0.304 wb	Z = 0.300 wb	Z = 0.299 wb	Z = 0.298 wb	Z = 0.297 wb

All dimensions in meters.

The average value of Z is very close to 0.3 wb.

To express vehicle combinations, z must be expressed in more general germs. From experiments which vehicle combinations it has been observed that the appropriate modified value of wb is the geometric mean of wb_e and wb_T, is

$$\frac{a+b}{2} = 0.3 \qquad \text{5-E}$$

Given that

$$a = \frac{z}{wb_e} \qquad \text{6-E}$$

$$b = \frac{z}{wb_T} \qquad 7\text{-E}$$

Therefore,

$$\frac{\frac{z}{wb_e} + \frac{z}{wb_T}}{2} = 0.3 \qquad 8\text{-E}$$

and

$$z = 0.6 \frac{wb_e \, wb_T}{wb_e + wb_T} \qquad 9\text{-E}$$

where as before wb_e = effective wheelbase

wb_T = total wheelbase of the vehicle combination.

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A Generalized Mathematical Solution for Transient Off Tracking of Single Vehicles and Truck Combinations

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1. Vehicles — off tracking — mathematical analysis — articulated vehicles
2. Tracking theory

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A simplified mathematical solution to predict transient off tracking of multiple vehicle combinations is presented. The generalized equations assume a constant radius curve of any angular duration with a tangent lead in and run out.

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